The Entropy Trajectory: A Perspective to Classify Complex Systems

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What are complex systems?

Having macroscopic properties which are not explicitly ascribed to the microscopic interactions between elements.

Empirically, the above definition is almost equal to
1) Difficult to predict the dynamics
2) Self-organized through long-term transient

Examples:
- Meteorological and oceanological systems
- Economical systems such as stock market
- Ecological systems such as animal population
Intelligence and complex systems

The organism is composed of millions of cells interacting one another, and an example of complex system.

In the organism, the microscopic interactions miraculously compose the intelligence, but complex systems are not always intelligent.

In an intelligent system, dynamics have a purpose
  • Human tries to maximize the profit
  • Even primitive bacteria try to survive and to prosper
Synthesizing complex systems is easy, but synthesizing intelligent one (Artificial life: ALife) is not possible so far.

What kind of complex systems can lead to intelligent capacity?

A view of ALife

Systems must adaptively abandon and renew their self-organized pattern.

Open question

pattern D

chaos

pattern A

chaos

pattern B

chaos

pattern C

chaos
Objective and methodology of current work

OBJECTIVE

• To establish a descriptor to check if a complex system in computation is able to abandon and to renew the self-organized pattern.

METHODOLOGY

• Creating various complex systems using 2-dimensional Cellular Automata (CAs).
• Time-series analysis of spatial and temporal entropy functions of the CAs to observe the self-organizing itinerancy.
Cellular Automata (CAs)

- CA was created by von Neumann and Ulam to simulate self-replication process of biological system.
- CA is a dynamical system defined on discrete space composed of ‘cells’.
- Each cell has an integer state which is updated every discrete time step.
- Update rule is local and homogenous.

**EXAMPLE**

The state assumes 0, 1, 2 or 3. (Total rule entry number is $4^9$.)

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Next state is maximum of 8 outer neighbors (periodic boundary condition)

<table>
<thead>
<tr>
<th>3</th>
<th>3</th>
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</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
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<td>3</td>
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<td>3</td>
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</tbody>
</table>

Time $t$ Time $t+1$
Spatial Pattern Entropy

\[ H_s(\tau) = -\sum_{k=1}^{16} P_s^k(\tau) \log_{16} (P_s^k(\tau)) \]

\( P_s^k(\tau) \) is the probability for a particular spatial pattern of local patch at the time step \( \tau \).
Temporal Pattern Entropy

\[ H_t(i, j) \equiv -\sum_{k} P_t^k(i, j) \log_4(P_t^k(i, j)) \]

\( P_t^k(i, j) \) is the probability for a particular temporal pattern of local patch at the cell \((i,j)\).
Classification of non-complex CA

Non-complex CAs have a fixed point entropy trajectory

static random spatial pattern (amorphous)

homogeneous or localized

No CA can visit.

chaos

$H_s$

$H_t$
Complex CA : GOL Type

\[ \lambda = \frac{N - n}{N} \]

- \( N \): Total rule entry number
- \( n \): Number of entries outputting zero state

**small \( \lambda \)-parameter**
- homogeneous or localized

**critical \( \lambda \)**
- unpredictable such as GOL

**large \( \lambda \)-parameter**
- chaotic

- Newly born non-zero cell
- Other non-zero cell
Complex CA : Crystal Type

\[ \mu = \frac{N - m}{N} \]

- \( N \): Total rule entry number
- \( m \): Number of entries not changing state

- small \( \mu \)-parameter
  - amorphous
- critical \( \mu \)
  - crystal-like
- large \( \mu \)-parameter
  - chaotic

- Newly born non-zero cell
- Other non-zero cell
Example of Complex CA (1): GOL Type

GOL

POND

\begin{align*}
H_s & 0.0 & 0.5 & 1.0 \\
H_t & 0.0 & 0.5 & 1.0
\end{align*}
Example of Complex CA (2): Crystal Type
Example of Complex CA (3): Crossover Type

StarWars by M. Wojtowicz
Concluding Remarks

Both GOL and Crystal types of CAs have simple entropy trajectory and do not renew their self-organized pattern.

The crossover type has an irregular entropy trajectory and do renew the self-organized pattern.

The entropy trajectory is a useful descriptor to measure the variety of self-organized patterns.

Could it be a descriptor of intelligence?