The entropy trajectory: A perspective to classify complex systems [1]

Japan Atomic Energy Research Institute Tomoaki SUZUDO [2]

Complex cellular automata (CAs) such as Conway's 'Game of Life' (GOL)[2] implied that some artificial dynamical systems could behave as if they were alive. However, one can notice the patterns developed in the GOL are different from truly living systems; especially, the living cells in the GOL do not construct intelligent functions with the sense of purpose. In living organism, some molecular construct solid components to keep its shape, others are used, e.g., for metabolism in liquid form. In addition, the solid components or whole individuals are to be destroyed and replaced by a new one when they get old. Exploiting these activities the living organisms ceaselessly change their strategy to survive. Obviously, such inherent capacities seem missing in the GOL. As far as the author knows, none of complex CAs are sufficient to be called a life-like system in this sense. In other words, those artificial systems are prevented from receiving limitless and complex development necessary to highly organized systems. Because this restriction is considered to be a key for enhancing the functionality of artificial life, I would like to discuss how this restriction can be identified by some numerical analysis of the CA's evolution.

The first important study of analyzing various CA's evolution was conducted by Wolfram[4]. He proposed four classes of one-dimensional CA and claimed that those belonging to the fourth class are most complex and probably capable of universal computation. Later, Langton used mutual information to characterize one-dimensional CAs[1], and revisited the Wolfram's Classes. He claimed that the Wolfram classes are parameterized by λ-parameter which measures a capacity of living cells to expand, and that complex CAs such as the GOL can be found by setting the rule to have a critical λ value between expansivity and stability. Thanks to their work, we know under what condition the complex CAs arise and how these CAs can be discriminated from others. However, we do not have means to classify different types of complex CAs, and cannot therefore identify the restriction of complexity described above.

In this study, I examined the detail of complexity of two-dimensional CAs applying the information entropy function which was used to characterize the crystallization of two-dimensional CAs[3]. Consider four adjacent sites of the CA space such as (i,j), $(i+1,j)$, $(i,j+1)$ and $(i+1,j+1)$; there are $2^4 = 16$ possible patterns for this local patch if each cell takes either 0 or 1 state. The entropy of the spatial pattern of CA configuration, H_s at an arbitrary time step τ can be defined by

$$H_s(\tau) \equiv -\sum_k P_s^k(\tau) log_{16}(P_s^k(\tau)), \tag{1}$$

where $P_s^k(\tau)$ is the probability for a particular pattern of the local patch at the time step τ. Note that a base of 16 is used for the logarithmic function as the entropy assumes a position between nil and unity. Consider a CA starting from random initial configuration (i.e. $H_s(0) \approx 1$), and calculate H_s in every single time step. The H_s for a non-complex CAs immediately converges into either very low or very high value, because they immediately evolve into homogeneous (or almost homogeneous) pattern or remain random. On the other hand, that of complex CA is expected to fluctuate between high and low values for a long period, because the pattern becomes regular or random at any time in the complex systems.

Similarly to H_s, it is possible to define the entropy associated with the temporal pattern. Consider an arbitrary site (i,j). If each cell takes either 0 or 1 state, there are four mapping patterns to the next time step per site, i.e. $0 \to 0$, $0 \to 1$, $1 \to 0$ and $1 \to 1$. From the probability of each mapping pattern, the entropy of the temporal CA development at the site of (i,j) during a certain

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[2] Tokai-mura, Naka-gun 319-1195 JAPAN @E-mail: suzudo@clsu3a0.tokai.jaeri.go.jp

short period can be defined by

$$H_t(i,j) \equiv -\sum_k P_t^k(i,j) log_4(P_t^k(i,j)), \quad (2)$$

where $P_t^k(i,j)$ is the probability for a updating particular pattern at (i,j) occurred during the period. The base of 4 is used for the logarithmic function as the entropy assumes a position between nil and unity. Similarly to H_s, H_t of non-complex CA quickly converges into either very low or very high value, and that of complex CAs is expected to fluctuate between high and low values for a long time, because the pattern change becomes regular or random at any time in the complex systems.

The actual data processing was conducted as follows. A CA was updated 100 times from an initial random configuration and all the history data were recorded. For this data set, the H_s averaged over this 100 spatial patterns and the H_t averaged over all cells' temporal patterns were calculated. Then, the CA was updated another 100 times for the next average H_t and H_s calculations. By repeating this process, the time-series data of H_t and H_s were given.

Figure 1 shows the result for the GOL; each entropy function change randomly, and the prediction of their behavior seems difficult. In this sense, the GOL is deserve to be complex. These two graphs, however, seem dependent on each other, see the $H_t - H_s$ trajectory in **Fig. 2**. This is not surprising itself, because the spatial pattern get random when the temporal change gets random. However, the further examination proved that the position of a line given by the trajectory rarely varies with the initial random configuration. In this sense, the behavior of the GOL is predictable. The same analyses for many other "complex" CAs showed that each CA had its specific trajectory, which is considered to be an indication of the artificial systems' restriction discussed above. Removing the restriction with some method should be a key to synthesize intelligent systems.

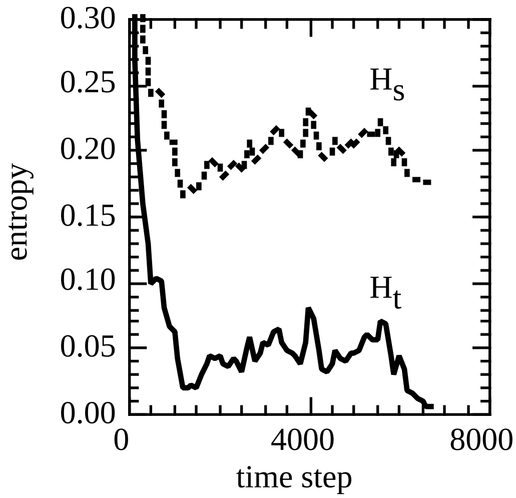

Fig. 1: The entropy functions of the GOL

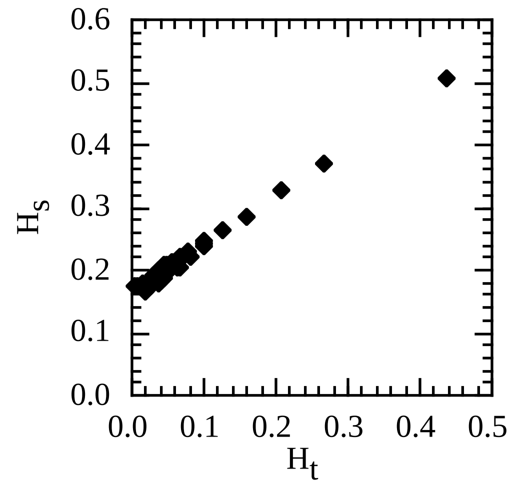

Fig. 2: The entropy trajectory of the GOL

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