

1.(5 点×4=20 点)

(1)

$$f_x = 18x^5 + 8x^3y^2 + 24x^2y^3 + 18xy^4, f_y = 4x^4y + 24x^3y^2 + 36x^2y^3 + 42y^5$$

(2)

$$f_x = \frac{3x^2 + 2xy + y^2}{2\sqrt{x^3 + x^2y + xy^2 + y^3}}, f_y = \frac{x^2 + 2xy + 3y^2}{2\sqrt{x^3 + x^2y + xy^2 + y^3}}$$

2.(5 点×6=30 点)

(1)

$$f_x = 6x^2 + 6xy + 5y^2 + 14xz + 2z^2 + 6yz \quad f_x = -2xe^{x^2+2y^2+3z^2} \sin(e^{x^2+2y^2+3z^2})$$

$$f_y = 9y^2 + 3x^2 + 10xy + 6yz + 5z^2 + 6xz \quad f_y = -4ye^{x^2+2y^2+3z^2} \sin(e^{x^2+2y^2+3z^2})$$

$$f_z = 12z^2 + 7x^2 + 4xz + 3y^2 + 10yz + 6xy \quad f_z = -6ze^{x^2+2y^2+3z^2} \sin(e^{x^2+2y^2+3z^2})$$

(2)

3. (2 点×5+4 点×10=50 点)

(1)

$$f_x = (2x + y + x^2 + xy + y^2)e^{x+y}, f_y = (x + 2y + x^2 + xy + y^2)e^{x+y},$$

$$f_{xx} = (2 + 4x + 2y + x^2 + xy + y^2)e^{x+y}, f_{xy} = (1 + 3x + 3y + x^2 + xy + y^2)e^{x+y}, f_{yy} = (2 + 2x + 4y + x^2 + xy + y^2)e^{x+y}$$

$$\textcircled{1} = 2x + y$$

$$\textcircled{2} = x + 2y$$

$$\textcircled{3} = 2 + 4x + 2y$$

$$\textcircled{4} = 1 + 3x + 3y$$

$$\textcircled{5} = 2 + 2x + 4y$$

(2)

$$f_x = \frac{1}{x+y} \sin(\sqrt{x^2+y^2}) + \frac{x}{\sqrt{x^2+y^2}} \log(x+y) \cos(\sqrt{x^2+y^2})$$

$$f_y = \frac{1}{x+y} \sin(\sqrt{x^2+y^2}) + \frac{y}{\sqrt{x^2+y^2}} \log(x+y) \cos(\sqrt{x^2+y^2})$$

$$f_{xx} = -(x+y)^{-2} \sin(\sqrt{x^2+y^2}) + (x+y)^{-1} x(x^2+y^2)^{-\frac{1}{2}} \cos(\sqrt{x^2+y^2}) + (x^2+y^2)^{-\frac{1}{2}} \log(x+y) \cos(\sqrt{x^2+y^2})$$

$$-x^2(x^2+y^2)^{-\frac{3}{2}} \log(x+y) \cos(\sqrt{x^2+y^2}) + \frac{x}{(x+y)\sqrt{x^2+y^2}} \cos(\sqrt{x^2+y^2})$$

$$-x(x^2+y^2)^{-\frac{1}{2}} \log(x+y) \sin(\sqrt{x^2+y^2}) x(x^2+y^2)^{-\frac{1}{2}}$$

$$\begin{aligned}
&= \{(x+y)^{-1}x(x^2+y^2)^{-\frac{1}{2}} + (x^2+y^2)^{-\frac{1}{2}}\log(x+y) \\
&\quad - x^2(x^2+y^2)^{-\frac{3}{2}}\log(x+y) + \frac{x}{(x+y)\sqrt{x^2+y^2}}\}\cos(\sqrt{x^2+y^2}) \\
&\quad + \{-(x+y)^{-2} - x(x^2+y^2)^{-\frac{1}{2}}x(x^2+y^2)^{-\frac{1}{2}}\log(x+y)\}\sin(\sqrt{x^2+y^2}) \\
&= \{2(x+y)^{-1}x(x^2+y^2)^{-\frac{1}{2}} + y^2(x^2+y^2)^{-\frac{3}{2}}\log(x+y)\}\cos(\sqrt{x^2+y^2}) \\
&\quad - \{(x+y)^{-2} + x^2(x^2+y^2)^{-1}\log(x+y)\}\sin(\sqrt{x^2+y^2}) \\
&= \{2(x+y)^{-1}x + y^2(x^2+y^2)^{-1}\log(x+y)\}(x^2+y^2)^{-\frac{1}{2}}\cos(\sqrt{x^2+y^2}) \\
&\quad - \{(x+y)^{-2} + x^2\log(x+y)(x^2+y^2)^{-1}\}\sin(\sqrt{x^2+y^2}) \\
&= \left\{\frac{2x}{x+y} + \frac{y^2\log(x+y)}{x^2+y^2}\right\}\frac{\cos(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}} - \left\{\frac{1}{(x+y)^2} + \frac{x^2\log(x+y)}{x^2+y^2}\right\}\sin(\sqrt{x^2+y^2})
\end{aligned}$$

$$\begin{aligned}
f_{xy} &= -(x+y)^{-2}\sin(\sqrt{x^2+y^2}) + (x+y)^{-1}y(x^2+y^2)^{-\frac{1}{2}}\cos(\sqrt{x^2+y^2}) + x\left(-\frac{1}{2}\right)(x^2+y^2)^{-\frac{3}{2}}2y\log(x+y)\cos(\sqrt{x^2+y^2}) \\
&\quad + \frac{x}{(x+y)\sqrt{x^2+y^2}}\cos(\sqrt{x^2+y^2}) - x(x^2+y^2)^{-\frac{1}{2}}\log(x+y)\sin(\sqrt{x^2+y^2})y(x^2+y^2)^{-\frac{1}{2}} \\
&= \{(x+y)^{-1}y(x^2+y^2)^{-\frac{1}{2}} - xy(x^2+y^2)^{-\frac{3}{2}}\log(x+y) + \frac{x}{(x+y)\sqrt{x^2+y^2}}\}\cos(\sqrt{x^2+y^2}) \\
&\quad + \{-(x+y)^{-2} - xy(x^2+y^2)^{-\frac{1}{2}}(x^2+y^2)^{-\frac{1}{2}}\log(x+y)\}\sin(\sqrt{x^2+y^2}) \\
&= \{(x^2+y^2)^{-\frac{1}{2}} - xy(x^2+y^2)^{-\frac{3}{2}}\log(x+y)\}\cos(\sqrt{x^2+y^2}) \\
&\quad - \{(x+y)^{-2} + xy(x^2+y^2)^{-1}\log(x+y)\}\sin(\sqrt{x^2+y^2}) \\
&= \left\{1 - \frac{xy\log(x+y)}{x^2+y^2}\right\}\frac{\cos(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}} - \left\{\frac{1}{(x+y)^2} + \frac{xy\log(x+y)}{x^2+y^2}\right\}\sin(\sqrt{x^2+y^2})
\end{aligned}$$

$$\begin{aligned}
f_{yy} &= -(x+y)^{-2}\sin(\sqrt{x^2+y^2}) + (x+y)^{-1}y(x^2+y^2)^{-\frac{1}{2}}\cos(\sqrt{x^2+y^2}) + (x^2+y^2)^{-\frac{1}{2}}\log(x+y)\cos(\sqrt{x^2+y^2}) \\
&\quad - y^2(x^2+y^2)^{-\frac{3}{2}}\log(x+y)\cos(\sqrt{x^2+y^2}) + \frac{y}{(x+y)\sqrt{x^2+y^2}}\cos(\sqrt{x^2+y^2}) \\
&\quad - y(x^2+y^2)^{-\frac{1}{2}}\log(x+y)\sin(\sqrt{x^2+y^2})y(x^2+y^2)^{-\frac{1}{2}} \\
&= \{(x+y)^{-1}y(x^2+y^2)^{-\frac{1}{2}} + (x^2+y^2)^{-\frac{1}{2}}\log(x+y) \\
&\quad - y^2(x^2+y^2)^{-\frac{3}{2}}\log(x+y) + \frac{y}{(x+y)\sqrt{x^2+y^2}}\}\cos(\sqrt{x^2+y^2}) \\
&\quad + \{-(x+y)^{-2} - y(x^2+y^2)^{-\frac{1}{2}}y(x^2+y^2)^{-\frac{1}{2}}\log(x+y)\}\sin(\sqrt{x^2+y^2})
\end{aligned}$$

$$\begin{aligned}
&= \{2(x+y)^{-1}y(x^2+y^2)^{-\frac{1}{2}} + x^2(x^2+y^2)^{-\frac{3}{2}}\log(x+y)\} \cos(\sqrt{x^2+y^2}) \\
&\quad - \{(x+y)^{-2} + y^2(x^2+y^2)^{-1}\log(x+y)\} \sin(\sqrt{x^2+y^2}) \\
&= \{2(x+y)^{-1}y + x^2(x^2+y^2)^{-1}\log(x+y)\}(x^2+y^2)^{-\frac{1}{2}} \cos(\sqrt{x^2+y^2}) \\
&\quad - \{(x+y)^{-2} + y^2\log(x+y)(x^2+y^2)^{-1}\} \sin(\sqrt{x^2+y^2}) \\
&= \left\{ \frac{2y}{x+y} + \frac{x^2 \log(x+y)}{x^2+y^2} \right\} \frac{\cos(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}} - \left\{ \frac{1}{(x+y)^2} + \frac{y^2 \log(x+y)}{x^2+y^2} \right\} \sin(\sqrt{x^2+y^2})
\end{aligned}$$

$$\textcircled{6} = \frac{1}{x+y}$$

$$\textcircled{7} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\textcircled{8} = \frac{1}{x+y}$$

$$\textcircled{9} = \frac{y}{\sqrt{x^2+y^2}}$$

$$\textcircled{10} = \frac{2x}{x+y} + \frac{y^2 \log(x+y)}{x^2+y^2}$$

$$\textcircled{11} = \frac{1}{(x+y)^2} + \frac{x^2 \log(x+y)}{x^2+y^2}$$

$$\textcircled{12} = 1 - \frac{xy \log(x+y)}{x^2+y^2}$$

$$\textcircled{13} = \frac{1}{(x+y)^2} + \frac{xy \log(x+y)}{x^2+y^2}$$

$$\textcircled{14} = \frac{2y}{x+y} + \frac{x^2 \log(x+y)}{x^2+y^2}$$

$$\textcircled{15} = \frac{1}{(x+y)^2} + \frac{y^2 \log(x+y)}{x^2+y^2}$$