数学 I　レポート No.2　解答　　　　　2017 年度

1.(10 点)

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\frac{dx}{dt}+\frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$=\{\frac{4x^3}{x^4+y^4}+(2x+y)\cos(x^2+xy+y^2)\}e^t+\{\frac{4y^3}{x^4+y^4}+(x+2y)\cos(x^2+xy+y^2)\}\frac{\cos t}{2+\sin t}$$

2.(5 点×2)

$$\frac{\partial z}{\partial u}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$$=e^{x+y}\sin(x^3+y^3)+3x^2e^{x+y}\cos(x^3+y^3)+\{e^{x+y}\sin(x^3+y^3)+3y^2e^{x+y}\cos(x^3+y^3)\}v$$

$$=(1+v)e^{x+y}\sin(x^3+y^3)+3(x^2+y^2v)e^{x+y}\cos(x^3+y^3)$$

$$\frac{\partial z}{\partial v}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial v}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

$$=e^{x+y}\sin(x^3+y^3)+3x^2e^{x+y}\cos(x^3+y^3)+\{e^{x+y}\sin(x^3+y^3)+3y^2e^{x+y}\cos(x^3+y^3)\}u$$

$$=(1+u)e^{x+y}\sin(x^3+y^3)+3(x^2+y^2u)e^{x+y}\cos(x^3+y^3)$$

3.(10 点)

$x^4+x^2y^2+y^4=x(4x^3\theta^3+2x\theta y^2\theta^2)+y(2x^2\theta^2y\theta+4y^3\theta^3)=4(x^4+x^2y^2+y^4)\theta^3$ より,

$4\theta^3=1$ ゆえに $\theta=\frac{1}{\sqrt[3]{4}}$

4.(10 点)

$$f(x,y)=f(0,0)+\{xf_x(0,0)+yf_y(0,0)\}+\frac{1}{2}\{x^2f_{xx}(0,0)+2xyf_{xy}(0,0)+y^2f_{yy}(0,0)\}+\cdots$$

から計算する.

$$f=\log(e^x+e^y-1),\ f_x=\frac{e^x}{e^x+e^y-1},\ f_y=\frac{e^y}{e^x+e^y-1},$$

$$f_{xx}=e^x(e^x+e^y-1)^{-1}-e^{2x}(e^x+e^y-1)^{-2},\ f_{xy}=-e^{x+y}(e^x+e^y-1)^{-2},$$

$$f_{yy}=e^y(e^x+e^y-1)^{-1}-e^{2y}(e^x+e^y-1)^{-2},$$

$$f(0,0)=0,\ f_x(0,0)=1,\ f_y(0,0)=1,\ f_{xx}(0,0)=0,\ f_{xy}(0,0)=-1,\ f_{yy}(0,0)=0$$

$$f(x,y)=0+(x+y)+\frac{1}{2}(0x^2-2xy+0y^2)+\cdots=x+y-xy+\cdots$$

5.(10 点)

$f=(x+y)(x^2+xy+y^2)^{-1}$,

$f_x=(x^2+xy+y^2)^{-1}-(x+y)(x^2+xy+y^2)^{-2}(2x+y)=-(x^2+2xy)(x^2+xy+y^2)^{-2}$,

$f_y=(x^2+xy+y^2)^{-1}-(x+y)(x^2+xy+y^2)^{-2}(x+2y)=-(2xy+y^2)(x^2+xy+y^2)^{-2}$,

$f_{xx}=-2(x+y)(x^2+xy+y^2)^{-2}+2(x^2+2xy)(x^2+xy+y^2)^{-3}(2x+y)$

$=2(x^3+3x^2y-y^3)(x^2+xy+y^2)^{-3}$,

$f_{xy}=-2y(x^2+xy+y^2)^{-2}-(2xy+y^2)(-2)(x^2+xy+y^2)^{-3}(2x+y)$

$=6xy(x+y)(x^2+xy+y^2)^{-3}$,

$f_{yy}=-(2x+2y)(x^2+xy+y^2)^{-2}-(2xy+y^2)(-2)(x^2+xy+y^2)^{-3}(x+2y)$

$=2(-x^3+3xy^2+y^3)(x^2+xy+y^2)^{-3}$,

$f(1,1)=\dfrac{2}{3}, f_x(1,1)=-\dfrac{1}{3}, f_y(1,1)=-\dfrac{1}{3}, f_{xx}(1,1)=\dfrac{2}{9}, f_{xy}(1,1)=\dfrac{4}{9}, f_{yy}(1,1)=\dfrac{2}{9}$,

$f(x,y)=\dfrac{2}{3}-\dfrac{1}{3}\{(x-1)+(y-1)\}+\dfrac{1}{9}\{(x-1)^2+4(x-1)(y-1)+(y-1)^2\}+\sim$

6.(10 点×2)

$$\frac{dy}{dx}=-\frac{f_x}{f_y}=-\frac{x^4+y}{y^4+x},\quad \frac{d^2y}{dx^2}=-\frac{f_{xx}f_y^2-2f_{xy}f_xf_y+f_{yy}f_x^2}{f_y^3}$$

$$=-\frac{4x^3(y^4+x)^2-2(x^4+y)(y^4+x)+4y^3(x^4+y)^2}{(y^4+x)^3}=-\frac{6xy(x^3y^3+2)}{(y^4+x)^3}$$

7. (10 点×2)

y,z を x の関数とみなし，両式を x で微分する．

$$\begin{cases}2x+2yy'+2zz'+1+y'+z'=0\\3x^2+3y^2y'+3z^2z'+3(1+y'+z')=0\end{cases}$$ これより，次式になる．

$$\begin{cases}(1+2y)y'+(1+2z)z'=-(1+2x)\\(1+y^2)y'+(1+z^2)z'=-(1+x^2)\end{cases}$$ この連立方程式にクラメルの公式を使う．

$$y'=\frac{\begin{vmatrix}-(1+2x) & 1+2z\\-(1+x^2) & 1+z^2\end{vmatrix}}{\begin{vmatrix}1+2y & 1+2z\\1+y^2 & 1+z^2\end{vmatrix}}=\frac{(x-z)(2xz+x+z-2)}{(z-y)(2yz+z+y-2)},\quad z'=\frac{\begin{vmatrix}1+2y & -(1+2x)\\1+y^2 & -(1+x^2)\end{vmatrix}}{\begin{vmatrix}1+2y & 1+2z\\1+y^2 & 1+z^2\end{vmatrix}}=\frac{(y-x)(2xy+x+y-2)}{(z-y)(2yz+z+y-2)}$$

8. (10 点)

$$J=\frac{\partial(x,y,z)}{\partial(u,v,w)}=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w}\\\frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w}\\\frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w}\end{vmatrix}=\begin{vmatrix}1 & 1 & 1\\v+w & u+w & u+v\\vw & uw & uv\end{vmatrix}$$

$$= uv(u+w) + vw(u+v) + uw(v+w) - vw(u+w) - uv(v+w) - uw(u+v)$$
$$= uv(u-v) + vw(v-w) + uw(w-u) = -(u-v)(v-w)(w-u)$$