1.(10 点×5)

(1)

$$I=\iint_D x^2y^3dxdy=\int_0^1dx\int_0^{x^2}x^2y^3dy=\int_0^1dxx^2\left[\frac{y^4}{4}\right]_0^{x^2}=\int_0^1\frac{x^{10}}{4}dx=\left[\frac{x^{11}}{4\cdot 11}\right]_0^1=\frac{1}{44}$$

(2)

$$I=\iint_D (x^4+2xy^3)dxdy=\int_0^1dy\int_0^{\sqrt{y}}(x^4+2xy^3)dx=\int_0^1dy\left[\frac{x^5}{5}+x^2y^3\right]_0^{\sqrt{y}}=\int_0^1(\frac{y^{\frac{5}{2}}}{5}+y^4)dy$$

$$=\left[\frac{y^{\frac{7}{2}}}{5\cdot\frac{7}{2}}+\frac{y^5}{5}\right]_0^1=\frac{9}{35}$$

(3)

$$I=\iint_D (x^3+3y^2)dxdy=\int_0^1dx\int_0^{1-x}(x^3+3y^2)dy=\int_0^1dx\left[x^3y+y^3\right]_0^{1-x}=\int_0^1\{x^3(1-x)+(1-x)^3\}dx$$

$1-x=t$　とおく.

$$I=\int_0^1(x^3-x^4)dx+\int_0^1(1-x)^3dx=\left[\frac{x^4}{4}-\frac{x^5}{5}\right]_0^1+\int_0^1t^3dt=\frac{1}{20}+\frac{1}{4}=\frac{3}{10}$$

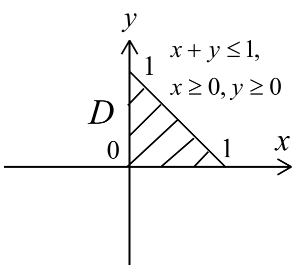

Fig.1-(3)

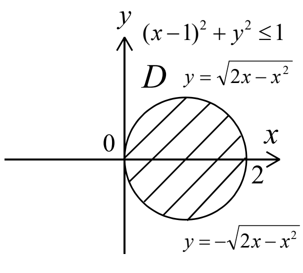

Fig.1-(4)

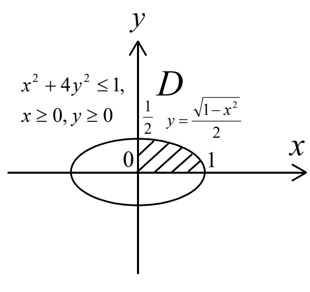

Fig.1-(5)

(4)

$$I=\iint_D x\sqrt{x}dxdy=\int_0^2dx\int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}}x\sqrt{x}dy=\int_0^2dxx\sqrt{x}\left[y\right]_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}}=\int_0^2 2x\sqrt{x}\sqrt{2x-x^2}dx$$

$$=2\int_0^2x^2\sqrt{2-x}dx$$

ここで

$2-x=t$ とおく． $dx=-dt$ である．

$$I=2\int_0^2 (t-2)^2\sqrt{t}dt=2\int_0^2 (t^{\frac{5}{2}}-4t^{\frac{3}{2}}+4t^{\frac{1}{2}})dt=2\left[\frac{t^{\frac{7}{2}}}{\frac{7}{2}}-\frac{4t^{\frac{5}{2}}}{\frac{5}{2}}+\frac{4t^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2=\frac{4}{7}\cdot 2^3\sqrt{2}-\frac{16}{5}\cdot 2^2\sqrt{2}+\frac{16}{3}\cdot 2\sqrt{2}$$

$$=2^5\sqrt{2}(\frac{1}{7}-\frac{2}{5}+\frac{1}{3})=\frac{256}{105}\sqrt{2}$$

(5)

$$I=\iint_D x^3ydxdy=\int_0^1 dx\int_0^{\frac{\sqrt{1-x^2}}{2}} x^3ydy=\int_0^1 dx\left[x^3\frac{y^2}{2}\right]_0^{\frac{\sqrt{1-x^2}}{2}}=\int_0^1\frac{x^3(1-x^2)}{8}dx=\frac{1}{8}\left[\frac{x^4}{4}-\frac{x^6}{6}\right]_0^1=\frac{1}{96}$$

2. (10 点×5)

(1)

$I=\iint_D (xy+x^2)dxdy$ 　　$\sqrt{x}+\sqrt{y}=1, y=(1-\sqrt{x})^2$ である．

$$I=\int_0^1 dx\int_0^{(1-\sqrt{x})^2}(xy+x^2)dy=\int_0^1 dx\left[\frac{xy^2}{2}+x^2y\right]_0^{(1-\sqrt{x})^2}=\int_0^1\{\frac{x(1-\sqrt{x})^4}{2}+x^2(1-\sqrt{x})^2\}dx$$

$t=\sqrt{x}$ とおく. 　　$x=t^2, dx=2tdt$

$$I=\int_0^1\{\frac{t^2(1-t)^4}{2}+t^4(1-t)^2\}2tdt=\int_0^1\{t^3(1-t)^4+2t^5(1-t)^2\}dt$$

$$=\int_0^1\{t^3(1-4t+6t^2-4t^3+t^4)+2t^5(1-2t+t^2)\}dt$$

$$=\int_0^1(t^3-4t^4+6t^5-4t^6+t^7+2t^5-4t^6+2t^7)dt$$

$$=\left[\frac{t^4}{4}-\frac{4}{5}t^5+t^6-\frac{4}{7}t^7+\frac{t^8}{8}+\frac{2}{6}t^6-\frac{4}{7}t^7+\frac{2}{8}t^8\right]_0^1$$

$$=\frac{1}{4}-\frac{4}{5}+1-\frac{4}{7}+\frac{1}{8}+\frac{1}{3}-\frac{4}{7}+\frac{1}{4}=\frac{13}{840}$$

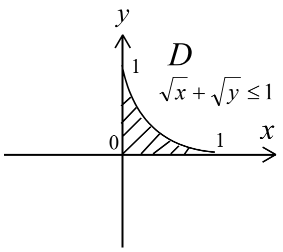

Fig.2-(1)

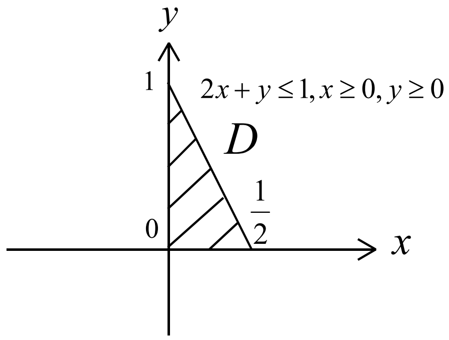

Fig.2-(2)

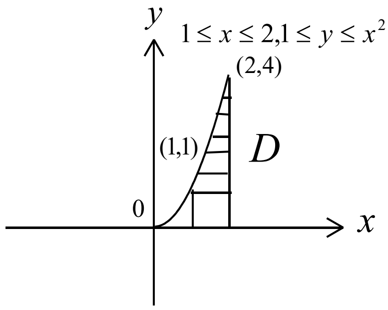

Fig.2-(3)

(2)

$$I=\iint_D(1-2x^2+3y^2)dxdy=\int_0^{\frac{1}{2}}dx\int_0^{1-2x}(1-2x^2+3y^2)dy=\int_0^{\frac{1}{2}}dx\Big[(1-2x^2)y+y^3\Big]_0^{1-2x}$$

$=\int_0^1\{(1-2x^2)(1-2x)+(1-2x)^3\}dx$　ここで$1-2x=t$とおく．$dx=-\frac{dt}{2}$である．

$$I=\int_1^0[\{1-2(\frac{1-t}{2})^2\}t+t^3](-\frac{dt}{2})=\frac{1}{2}\int_0^1(\frac{t^3}{2}+t^2+\frac{t}{2})dt=\frac{1}{2}(\frac{1}{8}+\frac{1}{3}+\frac{1}{4})=\frac{17}{48}$$

(3)

$$I=\iint_D\frac{x+y}{xy}dxdy=\int_1^2dx\int_1^{x^2}(\frac{1}{x}+\frac{1}{y})dy=\int_1^2dx\left[\frac{y}{x}+\log y\right]_1^{x^2}=\int_1^2(x+2\log x-\frac{1}{x})dx$$

$$=\left[\frac{x^2}{2}+2(x\log x-x)-\log x\right]_1^2=2+2(2\log 2-2)-\log 2-\frac{1}{2}-(-2)=3\log 2-\frac{1}{2}$$

(4)

$$I=\iiint_D(x+y+z+xyz)dxdydz=\int_0^1dx\int_0^{1-x}dy\int_0^{1-x-y}(x+y+z+xyz)dz$$

$$=\int_0^1dx\int_0^{1-x}\left[xz+yz+\frac{z^2}{2}+\frac{xyz^2}{2}\right]_0^{1-x-y}dy=\int_0^1dx\int_0^{1-x}\{(x+y)(1-x-y)+\frac{1}{2}(1+xy)(1-x-y)^2\}dy$$

$=\int_0^1dx\int_0^{1-x}\{(x+y)(1-x-y)+\frac{1}{2}(1+xy)(1-x-y)^2\}dy,\quad t=1-x-y$とおく．$dy=-dt$

$$=\int_0^1dx\int_{1-x}^0\{(x+1-x-t)t+\frac{1}{2}(1+x-x^2-xt)t^2\}(-dt)$$

$$=\int_0^1dx\int_0^{1-x}\{t+\frac{1}{2}(-1+x-x^2)t^2-\frac{1}{2}xt^3\}dt=\int_0^1dx\left[\frac{t^2}{2}+\frac{1}{6}(-1+x-x^2)t^3-\frac{1}{8}xt^4\right]_0^{1-x}$$

$=\int_0^1\{\frac{1}{2}(1-x)^2+\frac{1}{6}(-1+x-x^2)(1-x)^3-\frac{1}{8}x(1-x)^4\}dx, 1-x=u$とおく．$du=-dx$

$$=\int_1^0 [\frac{1}{2}u^2+\frac{1}{6}\{-u-(1-u)^2\}u^3-\frac{1}{8}(1-u)u^4](-du)=\int_0^1\{\frac{1}{2}u^2+\frac{1}{6}(-u^5+u^4-u^3)-\frac{1}{8}u^4+\frac{1}{8}u^5\}du$$

$$=\int_0^1(\frac{1}{2}u^2-\frac{1}{6}u^3+\frac{1}{24}u^4-\frac{1}{24}u^5)du=\frac{1}{6}-\frac{1}{24}+\frac{1}{120}-\frac{1}{144}=\frac{91}{720}$$

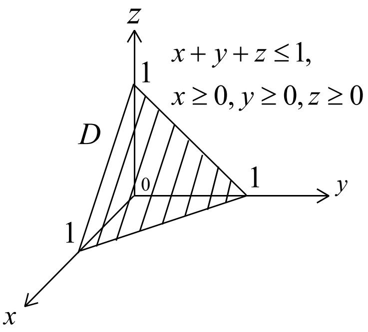

Fig.2-(4)

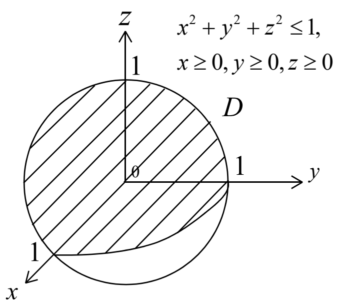

Fig.2-(5)

(5)

$D: x^2+y^2+z^2\leq 1, x\geq 0, y\geq 0, z\geq 0$ の領域から x, y, z の範囲を求める.

$z^2\leq 1-x^2-y^2, z\geq 0$ より $0\leq z\leq\sqrt{1-x^2-y^2}$ である. $1-x^2-y^2\geq 0, y\geq 0$ より

$0\leq y\leq\sqrt{1-x^2}$ である. また $1-x^2\geq 0, x\geq 0$ より $0\leq x\leq 1$ となる.

$$I=\iiint_D xzdxdydz=\int_0^1 dx\int_0^{\sqrt{1-x^2}}dy\int_0^{\sqrt{1-x^2-y^2}}xzdz=\int_0^1 dx\int_0^{\sqrt{1-x^2}}dy\left[\frac{xz^2}{2}\right]_0^{\sqrt{1-x^2-y^2}}$$

$$=\int_0^1 dx\int_0^{\sqrt{1-x^2}}\frac{x(1-x^2-y^2)}{2}dy=\int_0^1 dx\left[\frac{x(1-x^2)}{2}y-\frac{xy^3}{6}\right]_0^{\sqrt{1-x^2}}=\int_0^1\{\frac{x(1-x^2)}{2}\sqrt{1-x^2}-\frac{x(1-x^2)\sqrt{1-x^2}}{6}\}dx$$

$=\int_0^1\frac{x(1-x^2)}{3}\sqrt{1-x^2}dx, 1-x^2=t$ とおく. $xdx=-\frac{1}{2}dt$

$$=\int_1^0-\frac{t^{\frac{3}{2}}}{6}dt=\frac{1}{6}\int_0^1 t^{\frac{3}{2}}dt=\frac{1}{15}$$