数学 I　レポート No.8　解答　　　　2017 年度

1.(2 点×4)

(1)

$$S=\iint_D dxdy \quad \text{より} \qquad ① \; =dxdy$$

(2)

$$V=\iiint_D dxdydz \quad \text{より} \qquad ② \; =dxdydz$$

(3)

$$V=\iint_D f(x,y)dxdy \quad \text{より} \qquad ③ \; =f(x,y)dxdy$$

(4)

$$V=\pi\int_a^b f^2(x)dx \quad \text{より} \qquad ④ \; =\pi f^2(x)dx$$

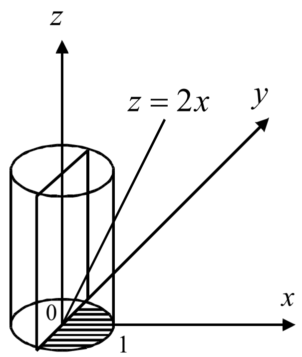
2.(10 点×2)

(1)

$z=2x$　である． xy 平面の正射影 D は $x^2+y^2\le 1, 0\le x\le 1$ の半円である．

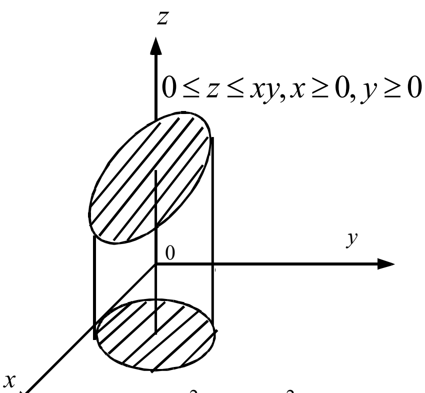
$$V=\iint_D 2xdxdy=2\int_0^1 dx\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} xdy=4\int_0^1 x\sqrt{1-x^2}dx \quad u=1-x^2 \text{ とおく．} \; -2xdx=du$$

$$V=4\int_1^0\sqrt{u}\left(-\frac{du}{2}\right)=2\int_0^1\sqrt{u}du=2\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1=\frac{4}{3}$$



$D:-\sqrt{1-x^2}\le y\le\sqrt{1-x^2}$,
$0\le x\le 1$

Fig.2-(1)



$D:(x-3)^2+(y-1)^2\le 1$

Fig.2-(2)

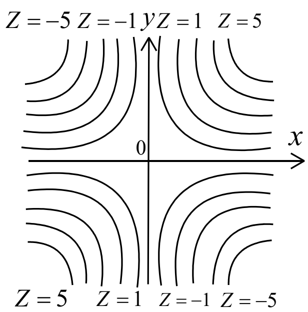


Fig.2-(2)-2

(2)

$D:(x-3)^2+(y-1)^2\le 1$ 　$,x-3=r\cos\theta, y-1=r\sin\theta$ とおく.

$x=r\cos\theta+3, y=r\sin\theta+1,\quad D':0\le r\le 1,0\le\theta\le 2\pi,\quad |J|=r$

$$V=\iint_D zdxdy=\iint_D xydxdy=\iint_{D'}(r\cos\theta+3)(r\sin\theta+1)rdrd\theta$$
$$=\int_0^1 dr\int_0^{2\pi}(r^3\cos\theta\sin\theta+r^2\cos\theta+3r^2\sin\theta+3r)d\theta$$
$$=\int_0^1 dr\int_0^{2\pi}(\frac{1}{2}r^3\sin 2\theta+r^2\cos\theta+3r^2\sin\theta+3r)d\theta=\int_0^1 dr\left[-\frac{r^3}{4}\cos 2\theta+r^2\sin\theta-3r^2\cos\theta+3r\theta\right]_0^{2\pi}$$
$$=6\pi\int_0^1 rdr=3\pi$$

3.(10 点×2)

(1)

$$V=\pi\int_0^{\pi}(\sin x+\frac{1}{3}\sin 3x)^2dx=\pi\int_0^{\pi}(\sin^2 x+\frac{2}{3}\sin x\sin 3x+\frac{1}{9}\sin^2 3x)dx$$
$$=\pi\int_0^{\pi}(\frac{1-\cos 2x}{2}+\frac{\cos 2x-\cos 4x}{3}+\frac{1-\cos 6x}{18})dx$$
$$=\pi\left[\frac{x-\frac{\sin 2x}{2}}{2}+\frac{\frac{\sin 2x}{2}-\frac{\sin 4x}{4}}{3}+\frac{x-\frac{\sin 6x}{6}}{18}\right]_0^{\pi}=\pi(\frac{\pi}{2}+\frac{\pi}{18})=\frac{5}{9}\pi^2$$

(2)

$$V=\pi\int_0^1(2x+x^3)^2dx=\pi\int_0^1(4x^2+4x^4+x^6)dx=\pi\left[\frac{4x^3}{3}+\frac{4}{5}x^5+\frac{1}{7}x^7\right]_0^1=(\frac{4}{3}+\frac{4}{5}+\frac{1}{7})\pi=\frac{239}{105}\pi$$

4.(4 点×3)

(1)

$S=\iint_D\sqrt{1+f_x^2+f_y^2}dxdy$ より　　　　①$=\sqrt{1+f_x^2+f_y^2}dxdy$

(2)

$S=\iint_{D'}\sqrt{r^2+(rf_r)^2+(f_\theta)^2}drd\theta$ より　②$=\sqrt{r^2+(rf_r)^2+(f_\theta)^2}drd\theta$

(3)

$S=2\pi\int_a^b f(x)\sqrt{1+f'(x)^2}dx$ より　　　　③$=2\pi f(x)\sqrt{1+f'(x)^2}dx$

5.(10 点×2)

(1)

xy 平面に対する正射影 D は $x^2+y^2\le 2x$ である．

これは $D:-\sqrt{2x-x^2}\le y\le\sqrt{2x-x^2},0\le x\le 2$ となる．

$z=f(x,y)=\sqrt{8x}$ から $f_x=\sqrt{\dfrac{2}{x}},f_y=0$ である．曲面積は上下の 2 組あるから，

$$S=2\iint_D\sqrt{1+f_x{}^2+f_y^2}dxdy=2\iint_D\sqrt{1+f_x{}^2}dxdy=2\iint_D\sqrt{1+\frac{2}{x}}dxdy=2\int_0^2dx\int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}}\sqrt{1+\frac{2}{x}}dy$$

$$=4\int_0^2\sqrt{1+\frac{2}{x}}\sqrt{2x-x^2}dx=4\int_0^2\sqrt{4-x^2}dx$$

$x=2\sin\theta$ とおく．　$dx=2\cos\theta d\theta$

$$S=16\int_0^{\frac{\pi}{2}}\cos^2\theta d\theta=16\int_0^{\frac{\pi}{2}}\frac{1+\cos 2\theta}{2}d\theta=8\left[\theta+\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}=4\pi$$

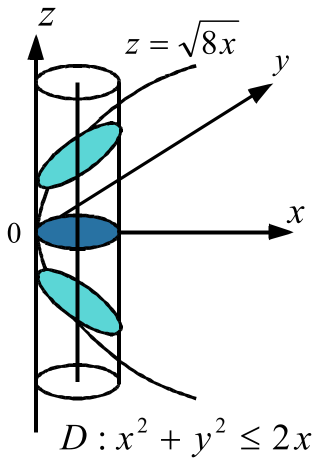


Fig.5-(1)

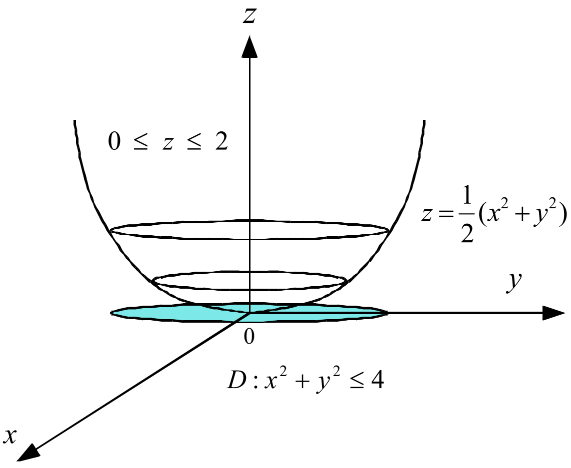


Fig.5-(2)

(2)

xy 平面に対する正射影 D は $0\le z\le 2,x^2+y^2=2z$ より，　$x^2+y^2\le 4$ である．
極座標変換 $(x=r\cos\theta,y=r\sin\theta)$ を使う．

$r^2=2z,z_r=r,z_\theta=0,D':0\le r\le 2,0\le\theta\le 2\pi$ である．

$$S=\iint_{D'}\sqrt{r^2+(rz_r)^2+(z_\theta)^2}\,drd\theta=\int_0^2 dr\int_0^{2\pi}d\theta\sqrt{r^2+r^4}=2\pi\int_0^2 r\sqrt{1+r^2}\,dr$$

$1+r^2=u$ とおく． $du=2rdr$ である．

$$S=2\pi\int_1^5\sqrt{u}\,\frac{du}{2}=\pi\int_1^5\sqrt{u}\,du=\pi\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^5=\frac{2}{3}\pi(5\sqrt{5}-1)$$

6.(10 点×2)

(1)

$y=2\sqrt{x},\ y'=\dfrac{1}{\sqrt{x}}$ である．

$$S=2\pi\int_0^1 y\sqrt{1+(y')^2}\,dx=2\pi\int_0^1 2\sqrt{x}\sqrt{1+\frac{1}{x}}\,dx=4\pi\int_0^1\sqrt{x+1}\,dx$$

$u=x+1$ とおく．

$$S=4\pi\int_1^2\sqrt{u}\,du=4\pi\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^2=\frac{8\pi}{3}(2\sqrt{2}-1)$$

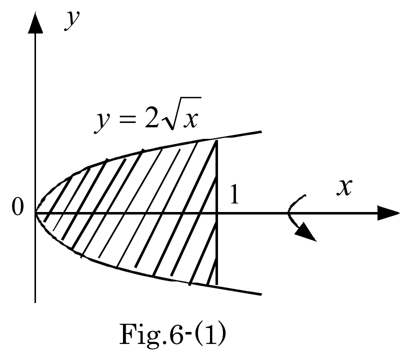


Fig.6-(1)

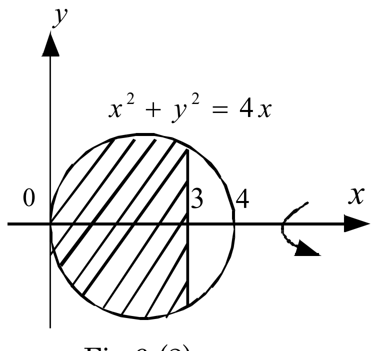


Fig.6-(2)

(2)

$x^2+y^2=4x,\ y\geq 0$　を考える． $y=\sqrt{4x-x^2},\ y'=\dfrac{(2-x)}{\sqrt{4x-x^2}}$ より

$$S=2\pi\int_0^3 y\sqrt{1+y'^2}\,dx=2\pi\int_0^3\sqrt{4x-x^2}\sqrt{1+\frac{(2-x)^2}{4x-x^2}}\,dx=2\pi\int_0^3\sqrt{(4x-x^2)+(2-x)^2}\,dx$$

$$=4\pi\int_0^3 dx=12\pi$$