2016.11.28

数学Ⅲ　　中間試験解答　　　　電気電子工学科

1. ($5点\times 2=10点$)

(1) $e^{\frac{\pi}{6}i}=\cos(\frac{\pi}{6})+i\sin(\frac{\pi}{6})=\frac{\sqrt{3}}{2}+\frac{1}{2}i$

(2) $8e^{\frac{\pi}{8}i}=8\{\cos(\frac{\pi}{8})+i\sin(\frac{\pi}{8})\}=8\{\sqrt{\frac{1+\cos(\frac{\pi}{4})}{2}}+i\sqrt{\frac{1-\cos(\frac{\pi}{4})}{2}}\}=8(\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}+i\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}})$

$=4(\sqrt{2+\sqrt{2}}+i\sqrt{2-\sqrt{2}})$

2. ($5点\times 2=10点$)

(1) $1+i\sqrt{3}=\sqrt{1+3}e^{\frac{\pi}{3}i}=2e^{\frac{\pi}{3}i}$

(2) $(\frac{1+i}{2})^2(\sqrt{3}-i)=(\frac{\sqrt{2}}{2}e^{\frac{\pi}{4}i})^2\sqrt{3+1}e^{-\frac{\pi}{6}i}=\frac{2}{2}e^{(\frac{\pi}{2}-\frac{\pi}{6})i}=e^{\frac{\pi}{3}i}$

3. ($5点\times 3=15点$)

$z^3=-8=8e^{\pi i}=8e^{\pi i+2n\pi i}, z=2e^{\frac{\pi}{3}i+\frac{2n\pi}{3}i}, (n=0,1,2)$

$z_0=2e^{\frac{\pi}{3}i}=2(\frac{1}{2}+i\frac{\sqrt{3}}{2})=1+i\sqrt{3}, z_1=2e^{\frac{3\pi}{3}i}=-2, z_2=2e^{\frac{5\pi}{3}i}=2(\frac{1}{2}-i\frac{\sqrt{3}}{2})=1-i\sqrt{3}$

$\underline{Ans.\ \ z=1+i\sqrt{3},\ -2,\ \ 1-i\sqrt{3}}$

4. ($6点\times 2=12点$)

$\left|z+\sqrt{3}\right|+\left|z-\sqrt{3}\right|=4, \left|x+iy+\sqrt{3}\right|+\left|x+iy-\sqrt{3}\right|=4, \sqrt{(x+\sqrt{3})^2+y^2}+\sqrt{(x-\sqrt{3})^2+y^2}=4,$

$(x+\sqrt{3})^2+y^2+(x-\sqrt{3})^2+y^2+2\sqrt{(x+\sqrt{3})^2+y^2}\sqrt{(x-\sqrt{3})^2+y^2}=16,$

$2(x^2+3+y^2)+2\sqrt{(x^2+3+y^2+2\sqrt{3}x)(x^2+3+y^2-2\sqrt{3}x)}=16,$

$\sqrt{(x^2+y^2+3)^2-12x^2}=8-(x^2+y^2+3), (x^2+y^2+3)^2-12x^2=(x^2+y^2+3)^2-16(x^2+y^2+3)+64,$

$4x^2+16y^2=16, \frac{x^2}{4}+y^2=1$

$\underline{Ans.\ \ \frac{x^2}{4}+y^2=1}$

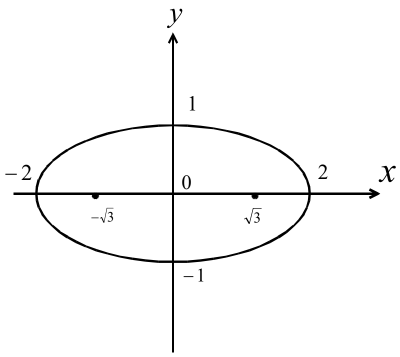

Fig.4

5. (5点×2＝10点)

(1) $$a_n=\frac{5^n}{7^n+1},\rho=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{\dfrac{5^n}{7^n+1}}{\dfrac{5^{n+1}}{7^{n+1}+1}}=\lim_{n\to\infty}\frac{5^n}{7^n+1}\frac{7^{n+1}+1}{5^{n+1}}=\lim_{n\to\infty}\frac{1}{5}\frac{7^{n+1}+1}{7^n+1}=\lim_{n\to\infty}\frac{1}{5}\frac{7+\dfrac{1}{7^n}}{1+\dfrac{1}{7^n}}=\frac{7}{5}$$

(2) $$a_n=\frac{(2n+3)!}{(3n+2)!},\rho=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{\dfrac{(2n+3)!}{(3n+2)!}}{\dfrac{(2n+5)!}{(3n+5)!}}=\lim_{n\to\infty}\frac{(2n+3)!}{(2n+5)!}\frac{(3n+5)!}{(3n+2)!}$$

$$=\lim_{n\to\infty}\frac{(3n+3)(3n+4)(3n+5)}{(2n+4)(2n+5)}=\infty$$

6. (2点×4)×2＝16点

(1) $$f(z)=\sin(\frac{\pi}{2}+2z+z^2),f'(z)=(2+2z)\cos(\frac{\pi}{2}+2z+z^2),$$

$$f''(z)=2\cos(\frac{\pi}{2}+2z+z^2)-(2+2z)^2\sin(\frac{\pi}{2}+2z+z^2),$$

$$f'''(z)=-2(2+2z)\sin(\frac{\pi}{2}+2z+z^2)-4(2+2z)\sin(\frac{\pi}{2}+2z+z^2)-(2+2z)^3\cos(\frac{\pi}{2}+2z+z^2)$$

$$f(0)=1,f'(0)=0,f''(0)=-4,f'''(0)=-4-8=-12$$

$$f(z)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}z^n=1+\frac{-4}{2!}z^2-\frac{12}{3!}z^3+\sim=1-2z^2-2z^3+\sim$$

(2) $f(z)=\log(1+3z+z^2), f'(z)=(3+2z)(1+3z+z^2)^{-1}$,

$f''(z)=2(1+3z+z^2)^{-1}-(3+2z)^2(1+3z+z^2)^{-2}$,

$f'''(z)=-2(3+2z)(1+3z+z^2)^{-2}-4(3+2z)(1+3z+z^2)^{-2}+2(3+2z)^3(1+3z+z^2)^{-3}$

$f(0)=0, f'(0)=3, f''(0)=2-9=-7, f'''(0)=-6-12+54=36$

$$f(z)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}z^n=\frac{3}{1!}z+\frac{-7}{2!}z^2+\frac{36}{3!}z^3+\sim=3z-\frac{7}{2}z^2+6z^3+\sim$$

7. (10点＋5点＝15点)

$$f(z)=\frac{1}{z^2+2z+5}=\frac{1}{(z-\lambda_1)(z-\lambda_2)}=\frac{\beta_1}{z-\lambda_1}+\frac{\beta_2}{z-\lambda_2},$$

$z^2+2z+5=0, \lambda_1,\lambda_2=-1\pm\sqrt{1-5}=-1\pm 2i, \lambda_1=-1+2i, \lambda_2=-1-2i,$

$$\beta_1=\lim_{z\to\lambda_1}(z-\lambda_1)\frac{1}{z^2+2z+5}=\lim_{z\to\lambda_1}\frac{1}{2z+2}=\frac{1}{2\lambda_1+2}=\frac{1}{4i},$$

$$\beta_2=\lim_{z\to\lambda_1}(z-\lambda_2)\frac{1}{z^2+2z+5}=\lim_{z\to\lambda_2}\frac{1}{2z+2}=\frac{1}{2\lambda_2+2}=-\frac{1}{4i}$$

$$f(z)=\frac{\beta_1}{z-\lambda_1}+\frac{\beta_2}{z-\lambda_2}=-\frac{\frac{\beta_1}{\lambda_1}}{1-\frac{z}{\lambda_1}}-\frac{\frac{\beta_2}{\lambda_2}}{1-\frac{z}{\lambda_2}}=-\sum_{n=0}^{\infty}\frac{\beta_1}{\lambda_1}\left(\frac{z}{\lambda_1}\right)^n-\sum_{n=0}^{\infty}\frac{\beta_2}{\lambda_2}\left(\frac{z}{\lambda_2}\right)^n$$

$$=-\sum_{n=0}^{\infty}\frac{1}{(-1+2i)4i}\left(\frac{1}{-1+2i}\right)^n z^n-\sum_{n=0}^{\infty}\frac{1}{(1+2i)4i}\left(\frac{1}{-1-2i}\right)^n z^n$$

$$=-\sum_{n=0}^{\infty}\frac{-1-2i}{5\times 4i}\left(\frac{-1-2i}{5}\right)^n z^n-\sum_{n=0}^{\infty}\frac{1-2i}{5\times 4i}\left(\frac{-1+2i}{5}\right)^n z^n$$

$$=\sum_{n=0}^{\infty}\frac{i}{4}\left(\frac{-1-2i}{5}\right)^{n+1}z^n-\sum_{n=0}^{\infty}\frac{i}{4}\left(\frac{-1+2i}{5}\right)^{n+1}z^n=\sum_{n=0}^{\infty}\frac{i}{4}\left\{\left(\frac{-1-2i}{5}\right)^{n+1}-\left(\frac{-1+2i}{5}\right)^{n+1}\right\}z^n$$

$$\rho_1=\lim_{n\to\infty}\left|\frac{\frac{i}{4}\left(\frac{-1-2i}{5}\right)^{n+1}}{\frac{i}{4}\left(\frac{-1-2i}{5}\right)^{n+2}}\right|=\left|\frac{1}{\frac{-1-2i}{5}}\right|=\left|\frac{5}{-1-2i}\right|=\frac{5}{\sqrt{5}}=\sqrt{5},$$

$$\rho_2=\lim_{n\to\infty}\left|\frac{\frac{i}{4}\left(\frac{-1+2i}{5}\right)^{n+1}}{\frac{i}{4}\left(\frac{-1+2i}{5}\right)^{n+2}}\right|=\left|\frac{1}{\frac{-1+2i}{5}}\right|=\left|\frac{5}{-1+2i}\right|=\frac{5}{\sqrt{5}}=\sqrt{5},$$

$\rho=\min(\rho_1,\rho_2)=\sqrt{5}$ *Ans.* $f(z)=\sum_{n=0}^{\infty}\frac{i}{4}\left\{\left(\frac{-1-2i}{5}\right)^{n+1}-\left(\frac{-1+2i}{5}\right)^{n+1}\right\}z^n, \rho=\sqrt{5}$

8. (6点 $\times 2 = 12$点)

(1) $\log(1 - i\sqrt{3}) = \log(\sqrt{1+3}e^{-\frac{\pi}{3}i + 2n\pi i}) = \log 2 + (-\frac{\pi}{3} + 2n\pi)i, \quad n \in Z$

(2) $\sin(3i) = (\frac{e^{i\theta} - e^{-i\theta}}{2i})_{\theta = 3i} = \frac{e^{-3} - e^{3}}{2i} = \frac{e^{3} - e^{-3}}{2} i = \sinh(3)i$