2016.11.30

数学Ⅲ　中間試験解答 情報科学科・応用生命システム工学科

1. (5点$\times 2=10$点)

(1) $e^{\frac{\pi}{3}i}=\cos(\frac{\pi}{3})+i\sin(\frac{\pi}{3})=\frac{1}{2}+\frac{\sqrt{3}}{2}i$

(2) $4e^{\frac{\pi}{12}i}=4\{\cos(\frac{\pi}{12})+i\sin(\frac{\pi}{12})\}=4\{\sqrt{\frac{1+\cos(\frac{\pi}{6})}{2}}+i\sqrt{\frac{1-\cos(\frac{\pi}{6})}{2}}\}$

$=4(\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}+i\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}})=4(\sqrt{\frac{2+\sqrt{3}}{4}}+i\sqrt{\frac{2-\sqrt{3}}{4}})=4(\sqrt{\frac{4+2\sqrt{3}}{8}}+i\sqrt{\frac{4-2\sqrt{3}}{8}})$

$=4(\frac{\sqrt{3}+1}{2\sqrt{2}}+i\frac{\sqrt{3}-1}{2\sqrt{2}})=(\sqrt{6}+\sqrt{2})+i(\sqrt{6}-\sqrt{2})$

2. (5点$\times 2=10$点)

(1) $\sqrt{3}+i=\sqrt{3+1}e^{\frac{\pi}{6}i}=2e^{\frac{\pi}{6}i}$

(2) $\frac{(1+i\sqrt{3})^4}{4}=\frac{(\sqrt{1+3}e^{\frac{\pi}{3}i})^4}{4}=\frac{2^4}{4}e^{\frac{4\pi}{3}i}=4e^{\frac{4\pi}{3}i}$

3. (5点$\times 3=15$点)

$z^3=i=e^{\frac{\pi}{2}i}=e^{\frac{\pi}{2}i+2n\pi i},z=e^{\frac{\pi}{6}i+\frac{2n\pi}{3}i},(n=0,1,2)$

$z_0=e^{\frac{\pi}{6}i}=\frac{\sqrt{3}}{2}+i\frac{1}{2},z_1=e^{\frac{5\pi}{6}i}=-\frac{\sqrt{3}}{2}+i\frac{1}{2},z_2=e^{\frac{9\pi}{6}i}=e^{\frac{3\pi}{2}i}=-i$

Ans. $z=\frac{\sqrt{3}}{2}+i\frac{1}{2},\quad -\frac{\sqrt{3}}{2}+i\frac{1}{2},\quad -i$

4. (6点$\times 2=12$点)

$|z+4|+|z-4|=10,|x+iy+4|+|x+iy-4|=10,\sqrt{(x+4)^2+y^2}+\sqrt{(x-4)^2+y^2}=10,$

$(x+4)^2+y^2+(x-4)^2+y^2+2\sqrt{(x+4)^2+y^2}\sqrt{(x-4)^2+y^2}=100,$

$$2(x^2+16+y^2)+2\sqrt{(x^2+16+y^2+8x)(x^2+16+y^2-8x)}=100,$$
$$\sqrt{(x^2+y^2+16)^2-64x^2}=50-(x^2+y^2+16),$$
$$(x^2+y^2+16)^2-64x^2=(x^2+y^2+16)^2-100(x^2+y^2+16)+2500,$$
$$36x^2+100y^2=900, \frac{x^2}{25}+\frac{y^2}{9}=1$$

$Ans.\ \ \frac{x^2}{25}+\frac{y^2}{9}=1$

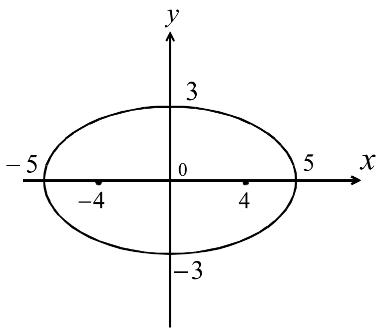

Fig.4

5. (5点×2＝10点)

(1) $$a_n=\frac{3^n}{8^n+1}, \rho=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{\frac{3^n}{8^n+1}}{\frac{3^{n+1}}{8^{n+1}+1}}=\lim_{n\to\infty}\frac{3^n}{8^n+1}\frac{8^{n+1}+1}{3^{n+1}}=\lim_{n\to\infty}\frac{1}{3}\frac{8^{n+1}+1}{8^n+1}=\lim_{n\to\infty}\frac{1}{3}\frac{8+\frac{1}{8^n}}{1+\frac{1}{8^n}}=\frac{8}{3}$$

(2) $$a_n=\frac{4^{-(2n+3)}}{n^4+5}, \rho=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{\frac{4^{-(2n+3)}}{n^4+5}}{\frac{4^{-(2n+5)}}{(n+1)^4+5}}=\lim_{n\to\infty}\frac{(n+1)^4+5}{n^4+5}\frac{4^{-(2n+3)}}{4^{-(2n+5)}}=\frac{4^5}{4^3}=16$$

6. (2点×4)×2＝16点

(1) $$f(z)=\cos(z+z^2), f'(z)=-(1+2z)\sin(z+z^2),$$
$$f''(z)=-2\sin(z+z^2)-(1+2z)^2\cos(z+z^2),$$
$$f'''(z)=-2(1+2z)\cos(z+z^2)-4(1+2z)\cos(z+z^2)+(1+2z)^3\sin(z+z^2),$$
$$f(0)=1, f'(0)=0, f''(0)=-1, f'''(0)=-2-4=-6$$
$$f(z)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}z^n=1+\frac{-1}{2!}z^2-\frac{6}{3!}z^3+\sim=1-\frac{1}{2}z^2-z^3+\sim$$

(2) $f(z)=e^{z+\sin z}, f'(z)=(1+\cos z)e^{z+\sin z},$

$f''(z)=-\sin z e^{z+\sin z}+(1+\cos z)^2 e^{z+\sin z},$

$f'''(z)=-\cos z e^{z+\sin z}-\sin z(1+\cos z)e^{z+\sin z}-2\sin z(1+\cos z)e^{z+\sin z}+(1+\cos z)^3 e^{z+\sin z}$

$f(0)=1, f'(0)=2, f''(0)=4, f'''(0)=-1+8=7$

$$f(z)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}z^n=1+\frac{2}{1!}z+\frac{4}{2!}z^2+\frac{7}{3!}z^3+\sim=1+2z+2z^2+\frac{7}{6}z^3+\sim$$

7. $(10$点$+5$点$=15$点$)$

$$f(z)=\frac{1}{z^2+2z+3}=\frac{1}{(z-\lambda_1)(z-\lambda_2)}=\frac{\beta_1}{z-\lambda_1}+\frac{\beta_2}{z-\lambda_2},$$

$z^2+2z+3=0, \lambda_1,\lambda_2=-1\pm\sqrt{1-3}=-1\pm\sqrt{2}i, \lambda_1=-1+\sqrt{2}i, \lambda_2=-1-\sqrt{2}i,$

$$\beta_1=\lim_{z\to\lambda_1}(z-\lambda_1)\frac{1}{z^2+2z+3}=\lim_{z\to\lambda_1}\frac{1}{2z+2}=\frac{1}{2\lambda_1+2}=\frac{1}{2\sqrt{2}i},$$

$$\beta_2=\lim_{z\to\lambda_1}(z-\lambda_2)\frac{1}{z^2+2z+3}=\lim_{z\to\lambda_2}\frac{1}{2z+2}=\frac{1}{2\lambda_2+2}=-\frac{1}{2\sqrt{2}i}$$

$$f(z)=\frac{\beta_1}{z-\lambda_1}+\frac{\beta_2}{z-\lambda_2}=-\frac{\frac{\beta_1}{\lambda_1}}{1-\frac{z}{\lambda_1}}-\frac{\frac{\beta_2}{\lambda_2}}{1-\frac{z}{\lambda_2}}=-\sum_{n=0}^{\infty}\frac{\beta_1}{\lambda_1}(\frac{z}{\lambda_1})^n-\sum_{n=0}^{\infty}\frac{\beta_2}{\lambda_2}(\frac{z}{\lambda_2})^n$$

$$=-\sum_{n=0}^{\infty}\frac{1}{(-1+\sqrt{2}i)2\sqrt{2}i}(\frac{1}{-1+\sqrt{2}i})^n z^n-\sum_{n=0}^{\infty}\frac{1}{(1+\sqrt{2}i)2\sqrt{2}i}(\frac{1}{-1-\sqrt{2}i})^n z^n$$

$$=-\sum_{n=0}^{\infty}\frac{(-1-\sqrt{2}i)\sqrt{2}}{3\times 4i}(\frac{-1-\sqrt{2}i}{3})^n z^n-\sum_{n=0}^{\infty}\frac{(1-\sqrt{2}i)\sqrt{2}}{3\times 4i}(\frac{-1+\sqrt{2}i}{3})^n z^n$$

$$=\sum_{n=0}^{\infty}\frac{i\sqrt{2}}{4}(\frac{-1-\sqrt{2}i}{3})^{n+1}z^n-\sum_{n=0}^{\infty}\frac{i\sqrt{2}}{4}(\frac{-1+\sqrt{2}i}{3})^{n+1}z^n=\sum_{n=0}^{\infty}\frac{i\sqrt{2}}{4}\{(\frac{-1-\sqrt{2}i}{3})^{n+1}-(\frac{-1+\sqrt{2}i}{3})^{n+1}\}z^n$$

$$\rho_1=\lim_{n\to\infty}\left|\frac{\frac{i\sqrt{2}}{4}(\frac{-1-\sqrt{2}i}{3})^{n+1}}{\frac{i\sqrt{2}}{4}(\frac{-1-\sqrt{2}i}{3})^{n+2}}\right|=\left|\frac{1}{\frac{-1-\sqrt{2}i}{3}}\right|=\left|\frac{3}{-1-\sqrt{2}i}\right|=\frac{3}{\sqrt{3}}=\sqrt{3},$$

$$\rho_2=\lim_{n\to\infty}\left|\frac{\frac{i\sqrt{2}}{4}(\frac{-1+\sqrt{2}i}{3})^{n+1}}{\frac{i\sqrt{2}}{4}(\frac{-1+\sqrt{2}i}{3})^{n+2}}\right|=\left|\frac{1}{\frac{-1+\sqrt{2}i}{3}}\right|=\left|\frac{3}{-1+\sqrt{2}i}\right|=\frac{3}{\sqrt{3}}=\sqrt{3},$$

$\rho=\min(\rho_1,\rho_2)=\sqrt{3}$ *Ans.* $f(z)=\sum_{n=0}^{\infty}\frac{i\sqrt{2}}{4}\{(\frac{-1-\sqrt{2}i}{3})^{n+1}-(\frac{-1+\sqrt{2}i}{3})^{n+1}\}z^n, \rho=\sqrt{3}$

8. $(6\text{点}\times 2=12\text{点})$

(1) $\log(\sqrt{3}+i)=\log(\sqrt{3+1}e^{\frac{\pi}{6}i+2n\pi i})=\log 2+(\frac{\pi}{6}+2n\pi)i,\quad n\in Z$

(2) $\cos(\frac{\pi}{4}+2i)=(\frac{e^{i\theta}+e^{-i\theta}}{2})_{\theta=\frac{\pi}{4}+2i}=\frac{e^{i(\frac{\pi}{4}+2i)}+e^{-i(\frac{\pi}{4}+2i)}}{2}=\frac{e^{-2}e^{i\frac{\pi}{4}}+e^{2}e^{-i\frac{\pi}{4}}}{2}=\frac{e^{-2}(1+i)+e^{2}(1-i)}{2\sqrt{2}}$

$=\frac{e^{-2}+e^{2}}{2\sqrt{2}}+i\frac{e^{-2}-e^{2}}{2\sqrt{2}}=\frac{\sqrt{2}}{4}(e^{2}+e^{-2})-i\frac{\sqrt{2}}{4}(e^{2}-e^{-2})=\frac{\sqrt{2}}{2}\cosh 2-i\frac{\sqrt{2}}{2}\sinh 2$