数学Ⅲ　レポート　No.2　　　解答　　　　　　　　(2016)

1.　(10 点×4=40 点)

(1)

$$a_n = \frac{3^n}{5^n + n},$$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{3^{n+1}}{5^{n+1}+(n+1)}}{\dfrac{3^n}{5^n+n}} = \frac{3^{n+1}}{5^{n+1}+(n+1)} \times \frac{5^n+n}{3^n} = \frac{3}{5+\dfrac{n}{5^n}+\dfrac{1}{5^n}} \times \frac{1+\dfrac{n}{5^n}}{1}$$

$$\overline{\lim_{n\to\infty}}\left(\frac{a_{n+1}}{a_n}\right) = \overline{\lim_{n\to\infty}}\left(\frac{3}{5+\dfrac{n}{5^n}+\dfrac{1}{5^n}} \times \frac{1+\dfrac{n}{5^n}}{1}\right) = \lim_{n\to\infty}\left(\frac{3}{5+\dfrac{n}{5^n}+\dfrac{1}{5^n}} \times \frac{1+\dfrac{n}{5^n}}{1}\right) = \frac{3}{5} < 1$$

<u>収束</u>

(2)

$$a_n = \frac{7^n}{3^{2n}+1},$$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{7^{n+1}}{3^{2(n+1)}+1}}{\dfrac{7^n}{3^{2n}+1}} = \frac{3^{2n}+1}{3^{2n+2}+1} \times \frac{7^{n+1}}{7^n} = \frac{1+\dfrac{1}{3^{2n}}}{9+\dfrac{1}{3^{2n}}} \times 7$$

$$\overline{\lim_{n\to\infty}}\left(\frac{a_{n+1}}{a_n}\right) = \overline{\lim_{n\to\infty}}\left(\frac{1+\dfrac{1}{3^{2n}}}{9+\dfrac{1}{3^{2n}}} \times 7\right) = \lim_{n\to\infty}\left(\frac{1+\dfrac{1}{3^{2n}}}{9+\dfrac{1}{3^{2n}}} \times 7\right) = \frac{7}{9} < 1$$

<u>収束</u>

(3)

$$a_n = \frac{e^n}{n^2+1},$$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{e^{n+1}}{(n+1)^2+1}}{\dfrac{e^n}{n^2+1}} = \frac{n^2+1}{(n+1)^2+1} \times \frac{e^{n+1}}{e^n} = \frac{1+\dfrac{1}{n^2}}{(1+\dfrac{1}{n})^2+\dfrac{1}{n^2}} \times e$$

$$\lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right) = \lim_{n\to\infty}\left(\frac{1+\dfrac{1}{n^2}}{(1+\dfrac{1}{n})^2+\dfrac{1}{n^2}} \times e\right) = e > 1$$

$$\varliminf_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)=\varliminf_{n\to\infty}\left(\frac{1+\frac{1}{n^2}}{(1+\frac{1}{n})^2+\frac{1}{n^2}}\times e\right)=\lim_{n\to\infty}\left(\frac{1+\frac{1}{n^2}}{(1+\frac{1}{n})^2+\frac{1}{n^2}}\times e\right)=e>1$$

<u>発散</u>

(4)

$$a_n=\frac{4^{-n}}{n^3+1},$$

$$\frac{a_{n+1}}{a_n}=\frac{\dfrac{4^{-n-1}}{(n+1)^3+1}}{\dfrac{4^{-n}}{n^3+1}}=\frac{4^{-n-1}}{4^{-n}}\times\frac{n^3+1}{(n+1)^3+1}=\frac{1}{4}\times\frac{n^3+1}{(n+1)^3+1}$$

$$\lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)=\lim_{n\to\infty}\left(\frac{1}{4}\times\frac{n^3+1}{(n+1)^3+1}\right)=\frac{1}{4}<1$$

$$\varlimsup_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)=\lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right)=\lim_{n\to\infty}\left(\frac{1}{4}\times\frac{n^3+1}{(n+1)^3+1}\right)=\frac{1}{4}<1$$

<u>収束</u>

2. (10 点×4=40 点)

(1)

$$a_n=\frac{1}{2^n+n^2},$$

$$\frac{a_n}{a_{n+1}}=\frac{\dfrac{1}{2^n+n^2}}{\dfrac{1}{2^{n+1}+(n+1)^2}}=\frac{2^{n+1}+(n+1)^2}{2^n+n^2}=\frac{2+\dfrac{(n+1)^2}{2^n}}{1+\dfrac{n^2}{2^n}}$$

$$\lim_{n\to\infty}\frac{n^2}{2^n}=\lim_{n\to\infty}\frac{2n}{2^n\log 2}=\lim_{n\to\infty}\frac{2}{2^n(\log 2)^2}=0,\lim_{n\to\infty}\frac{(n+1)^2}{2^n}=0$$

$$\rho=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\left(\frac{2+\dfrac{(n+1)^2}{2^n}}{1+\dfrac{n^2}{2^n}}\right)=2$$

(2)

$$a_n=\frac{1}{(n+2)\log(n^2+2)},$$

$$\frac{a_n}{a_{n+1}}=\frac{\dfrac{1}{(n+2)\log(n^2+2)}}{\dfrac{1}{(n+3)\log\{(n+1)^2+2\}}}=\frac{(n+3)\log\{(n+1)^2+2\}}{(n+2)\log(n^2+2)}=\frac{(n+3)}{(n+2)}\times\frac{\log\{(n+1)^2+2\}}{\log(n^2+2)}$$

$$\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{(n+3)}{(n+2)}\times\frac{\log\{(n+1)^2+2\}}{\log(n^2+2)}=\lim_{n\to\infty}\frac{\log\{(n+1)^2+2\}}{\log(n^2+2)}=\lim_{n\to\infty}\frac{\dfrac{2(n+1)}{(n+1)^2+2}}{\dfrac{2n}{n^2+2}}$$

$$\rho=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\{\frac{2(n+1)}{2n}\times\frac{n^2+2}{(n+1)^2+2}\}=1$$

(3)

$$a_n=\frac{(n+3)!}{(2n+1)!},$$

$$\frac{a_n}{a_{n+1}}=\frac{\dfrac{(n+3)!}{(2n+1)!}}{\dfrac{(n+4)!}{(2n+3)!}}=\frac{(2n+3)!}{(2n+1)!}\times\frac{(n+3)!}{(n+4)!}=\frac{(2n+2)(2n+3)}{(n+4)}$$

$$\rho=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{(2n+2)(2n+3)}{(n+4)}=\infty$$

(4)

$$a_n=\frac{4^n}{n^4+1},$$

$$\rho=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\frac{\dfrac{4^n}{n^4+1}}{\dfrac{4^{n+1}}{(n+1)^4+1}}=\lim_{n\to\infty}\frac{4^n}{4^{n+1}}\times\frac{(n+1)^4+1}{n^4+1}=\lim_{n\to\infty}\frac{1}{4}\times\frac{(1+\frac{1}{n})^4+\frac{1}{n^4}}{1+\frac{1}{n^4}}=\frac{1}{4}$$

3. (10 点×2=20 点)

(1)

$$a_n=2^{(3n-1)},$$

$$\frac{1}{\rho}=\overline{\lim_{n\to\infty}}\sqrt[n]{|a_n|}=\overline{\lim_{n\to\infty}}2^{\frac{3n-1}{n}}=\lim_{n\to\infty}2^3=8,\rho=\frac{1}{8}$$

(2)

$$a_n = \frac{3^{-(7n^2+5n+3)}}{n^2+2},$$

$$\frac{1}{\rho} = \overline{\lim_{n\to\infty}} \sqrt[n]{|a_n|} = \overline{\lim_{n\to\infty}} \sqrt[n]{\frac{3^{-(7n^2+5n+3)}}{n^2+2}} = \lim_{n\to\infty} \frac{3^{-\frac{7n^2+5n+3}{n}}}{(n^2+2)^{\frac{1}{n}}}$$

$$w = (n^2+2)^{\frac{1}{n}}, \log w = \frac{1}{n}\log(n^2+2)$$

$$\lim_{n\to\infty} \log w = \lim_{n\to\infty} \frac{\log(n^2+2)}{n} = \lim_{n\to\infty} \frac{\frac{2n}{n^2+2}}{1} = \lim_{n\to\infty} \frac{2n}{n^2+2} = 0$$

$$\lim_{n\to\infty} w = 1$$

$$\frac{1}{\rho} = \lim_{n\to\infty} \frac{3^{-\frac{7n^2+5n+3}{n}}}{w} = \lim_{n\to\infty} \frac{3^{-7n-5-\frac{3}{n}}}{w} = \frac{0}{1} = 0, \rho = \infty$$