数学Ⅲ　レポート　No.4　　解答　　　　　(2016)

1.　(10 点×2=20 点)

(1)

$$u = x^2 - y^2 + 3x, v = 2xy + 3y$$

$$\frac{\partial u}{\partial x} = 2x + 3, \frac{\partial v}{\partial y} = 2x + 3, \therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -2y, \frac{\partial v}{\partial x} = 2y, \therefore \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

正則

(2)

$$u = x^3 - 3y^2x + 2x^2 - 2y^2 + y, v = 3x^2y - y^3 + 4xy + x$$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 4x, \frac{\partial v}{\partial y} = 3x^2 - 3y^2 + 4x, \therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y},$$

$$\frac{\partial u}{\partial y} = -6xy - 4y + 1, \frac{\partial v}{\partial x} = 6xy + 4y + 1, \therefore \frac{\partial u}{\partial y} \neq -\frac{\partial v}{\partial x}$$

正則でない

2.　(10 点×2=20 点)

(1)

$$u = x^4 - 6x^2y^2 + y^4 + x^3 - 3xy^2,$$

$$\frac{\partial u}{\partial x} = 4x^3 - 12xy^2 + 3x^2 - 3y^2 = \frac{\partial v}{\partial y}, v = \int (4x^3 - 12xy^2 + 3x^2 - 3y^2)dy = 4x^3y - 4xy^3 + 3x^2y - y^3 + C_1(x),$$

$$\frac{\partial u}{\partial y} = -12x^2y + 4y^3 - 6xy = -\frac{\partial v}{\partial x} = -12x^2y + 4y^3 - 6xy - C_1'(x), \therefore C_1'(x) = 0, C_1(x) = C,$$

$$f(z) = x^4 - 6x^2y^2 + y^4 + x^3 - 3xy^2 + i(4x^3y - 4xy^3 + 3x^2y - y^3 + C), f(0) = C = 0,$$

$$f(z) = x^4 - 6x^2y^2 + y^4 + x^3 - 3xy^2 + i(4x^3y - 4xy^3 + 3x^2y - y^3)$$

$$z = x + iy, \bar{z} = x - iy, x = z - iy, f(z) = u(x,y) + iv(x,y) = u(z - iy, y) + iv(z - iy, y),$$

*Ans.* $\underline{v(x,y) = 4x^3y - 4xy^3 + 3x^2y - y^3, f(z) = u(z,0) + iv(z,0) = z^4 + z^3}$

(2)

$$u(x,y) = x\sin(x)\cosh(y) - y\cos(x)\sinh(y),$$

$$\frac{\partial u}{\partial x} = \sin(x)\cosh(y) + x\cos(x)\cosh(y) + y\sin(x)\sinh(y) = \frac{\partial v}{\partial y},$$

$$v = \int \{\sin(x)\cosh(y) + x\cos(x)\cosh(y) + y\sin(x)\sinh(y)\}dy,$$

$$\int \cosh(y)dy = \int \frac{e^y + e^{-y}}{2}dy = \frac{e^y - e^{-y}}{2} + C = \sinh(x) + C, \int \sinh(y)dy$$

$$= \int \frac{e^y - e^{-y}}{2}dy = \frac{e^y + e^{-y}}{2} + C = \cosh(x) + C,$$

$\int y\sinh(y)dy, u_1 = y, \dot{v}_1 = \sinh(y), \dot{u}_1 = 1, v_1 = \cosh(y), \int y\sinh(y)dy$

$= u_1 v_1 - \int \dot{u}_1 v_1 dy = y\cosh(y) - \sinh(y)$

$v = \sin(x)\sinh(y) + x\cos(x)\sinh(y) + y\sin(x)\cosh(y) - \sin(x)\sinh(y) + C_1(x)$

$= x\cos(x)\sinh(y) + y\sin(x)\cosh(y) + C_1(x)$

$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}, \frac{\partial v}{\partial x} = \cos(x)\sinh(y) - x\sin(x)\sinh(y) + y\cos(x)\cosh(y) + C_1'(x),$

$\frac{\partial u}{\partial y} = x\sin(x)\sinh(y) - \cos(x)\sinh(y) - y\cos(x)\cosh(y), \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ より $C_1'(x) = 0, C_1(x) = C$

$f(z) = x\sin(x)\cosh(y) - y\cos(x)\sinh(y) + i\{x\cos(x)\sinh(y) + y\sin(x)\cosh(y) + C\},$

$f(0) = u(0,0) + iv(0,0) = iC = 0$ より $C = 0$ $v = x\cos(x)\sinh(y) + y\sin(x)\cosh(y)$

$z = x + iy, \bar{z} = x - iy, x = z - iy, f(z) = u(x,y) + iv(x,y) = u(z-iy, y) + iv(z-iy, y),$

$f(z) = x\sin(x)\cosh(y) - y\cos(x)\sinh(y) + i\{x\cos(x)\sinh(y) + y\sin(x)\cosh(y)\},$

$f(z) = u(z,0) + iv(z,0) = z\sin(z)$ *Ans.* $\underline{v = x\cos(x)\sinh(y) + y\sin(x)\cosh(y), f(z) = z\sin(z)}$

3. (10点 $\times 2 = 20$点)

(1)

$v = -\sin(2x)\sinh(2y)$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$\frac{\partial v}{\partial y} = -2\sin(2x)\cosh(2y), u = -\int 2\sin(2x)\cosh(2y)dx = \cos(2x)\cosh(2y) + C_1(y),$

$\frac{\partial u}{\partial y} = 2\cos(2x)\sinh(2y) + C_1'(y) = -\frac{\partial v}{\partial x} = 2\cos(2x)\sinh(2y)$ より $C_1'(y) = 0, C_1(y) = C$

$f(z) = u + iv = \cos(2x)\cosh(2y) + C + i\{-\sin(2x)\sinh(2y)\}, f(0) = 1 + C = 1, C = 0$

$f(z) = \cos(2x)\cosh(2y) + i\{-\sin(2x)\sinh(2y)\}, f(z) = u(z,0) + iv(z,0) = \cos(2z)$

*Ans.* $\underline{u = \cos(2x)\cosh(2y), f(z) = \cos(2z)}$

(2)

$v = \arctan(\frac{y}{1+x}), f(0) = 0$

$\frac{y}{1+x} = \tan(v), \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}, \frac{y}{1+x} = \tan(v)$の両辺を$y$で偏微分する. $\frac{1}{1+x} = \sec^2 v \frac{\partial v}{\partial y}$

$$\frac{\partial v}{\partial y}=\frac{1}{(1+x)\sec^2 v}=\frac{\partial u}{\partial x},\sec^2 v=1+\tan^2 v=1+(\frac{y}{1+x})^2,\frac{\partial u}{\partial x}=\frac{1}{(1+x)\{1+(\frac{y}{1+x})^2\}}=\frac{1+x}{(1+x)^2+y^2}$$

$w=(1+x)^2$とおく. $2(1+x)dx=dw,\quad u=\int\frac{1+x}{(1+x)^2+y^2}dx=\frac{1}{2}\int\frac{dw}{w+y^2}=\frac{1}{2}\log(w+y^2)+C_1(y),$

$u=\frac{1}{2}\log\{(1+x)^2+y^2\}+C_1(y),\frac{\partial u}{\partial y}=\frac{1}{2}\times\frac{2y}{(1+x)^2+y^2}+C_1'(y),$一方 $\frac{\partial}{\partial x}\frac{y}{1+x}=\frac{\partial}{\partial x}\tan(v),$

$$-\frac{y}{(1+x)^2}=\sec^2 v\frac{\partial v}{\partial x},\frac{\partial v}{\partial x}=-\frac{y}{(1+x)^2\sec^2 v}=-\frac{y}{(1+x)^2(1+\tan^2 v)}=-\frac{y}{(1+x)^2\{1+y^2(1+x)^{-2}\}}$$

$=-\frac{y}{(1+x)^2+y^2},\frac{\partial u}{\partial y}=\frac{y}{(1+x)^2+y^2}+C_1'(y)=\frac{y}{(1+x)^2+y^2}$ より $C_1'(y)=0,\quad C_1(y)=C,$

$$u=\frac{1}{2}\log\{(1+x)^2+y^2\}+C,f(z)=u+iv=\frac{1}{2}\log\{(1+x)^2+y^2\}+C+i\arctan(\frac{y}{1+x}),f(0)=C=0,$$

$$f(z)=\frac{1}{2}\log\{(1+x)^2+y^2\}+i\arctan(\frac{y}{1+x}),z=x+iy,x=z-iy,$$

$f(z)=\frac{1}{2}\log\{(1+z-iy)^2+y^2\}+i\arctan(\frac{y}{1+z-iy}),y=0$ とおく. $f(z)=\frac{1}{2}\log(1+z)^2=\log(1+z)$

*Ans.* $\underline{u=\frac{1}{2}\log\{(1+x)^2+y^2\},f(z)=\log(1+z)}$

4. $(5$点$\times 4)\times 2=40$点

(1)

$$f(z)=(x^3-3xy^2+k_1x^2-3y^2+5x)+i(3x^2y-y^3+k_2xy+k_3y)=u+iv$$

$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y},\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$ より $\frac{\partial u}{\partial x}=3x^2-3y^2+2k_1x+5=\frac{\partial v}{\partial y}=3x^2-3y^2+k_2x+k_3,$

$$2k_1=k_2,5=k_3,\frac{\partial u}{\partial y}=-6xy-6y=-\frac{\partial v}{\partial x}=-(6xy+k_2)=-6xy-k_2y,k_2=6,$$

$$\therefore k_1=3,k_2=6,k_3=5,f(z)=(x^3-3xy^2+3x^2-3y^2+5x)+i(3x^2y-y^3+6xy+5y),$$

$$f(z)=u(x,y)+iv(x,y)=u(z-iy,y)+iv(z-iy,y),f(z)=u(z,0)+iv(z,0)=z^3+3z^2+5z$$

*Ans.* $\underline{k_1=3,k_2=6,k_3=5,\quad f(z)=z^3+3z^2+5z}$

(2)

$$f(z)=\frac{2x+k_2}{(x+k_1)^2+y^2}+i\frac{k_3y}{(x+k_1)^2+y^2}=u+iv,f(0)=2$$

$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y},\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$ より $\frac{\partial u}{\partial x}=\frac{\partial}{\partial x}\{\frac{2x+k_2}{(x+k_1)^2+y^2}\}=\frac{2\{(x+k_1)^2+y^2\}-(2x+k_2)\times 2(x+k_1)}{\{(x+k_1)^2+y^2\}^2}$

$$=\frac{\partial v}{\partial y}=\frac{\partial}{\partial y}\{\frac{k_3y}{(x+k_1)^2+y^2}\}=\frac{k_3\{(x+k_1)^2+y^2\}-2k_3y^2}{\{(x+k_1)^2+y^2\}^2},$$

$$2\{(x+k_1)^2+y^2\}-(2x+k_2)\times 2(x+k_1)=k_3\{(x+k_1)^2+y^2\}-2k_3y^2,$$

$$2(x^2+2k_1x+k_1{}^2+y^2)-2\{2x^2+(2k_1+k_2)x+k_1k_2\}=k_3(x^2+2k_1x+k_1{}^2+y^2)-2k_3y^2,$$

$$-2x^2-2k_2x+(2k_1{}^2-2k_1k_2)+2y^2=k_3x^2+2k_1k_3x+k_3k_1{}^2-k_3y^2,$$

$\therefore -2=k_3, -2k_2=2k_1k_3, (2k_1{}^2-2k_1k_2)=k_3k_1{}^2, 2=-k_3$ より$k_3=-2, k_2=2k_1, 4k_1^2=2k_1k_2$

$$f(z)=\frac{2x+2k_1}{(x+k_1)^2+y^2}+i\frac{-2y}{(x+k_1)^2+y^2}, f(0)=\frac{2k_1}{k_1^2}=2, k_1\neq 0, k_1=1, k_2=2, k_3=-2$$

$$f(z)=\frac{2x+2}{(x+1)^2+y^2}+i\frac{-2y}{(x+1)^2+y^2}, f(z)=u(x,y)+iv(x,y)=u(z-iy,y)+iv(z-iy,y),$$

$$f(z)=u(z,0)+iv(z,0)=\frac{2(z+1)}{(z+1)^2}=\frac{2}{z+1}$$

*Ans.* $k_1=1, k_2=2, k_3=-2,\ \ f(z)=\dfrac{2}{z+1}$