数学Ⅲ　レポート　No.6　　解答　　(2016)

1.　(20 点×5=100 点)

(1)

$$J=\int_{-\infty}^{\infty}\frac{1}{x^2+2x+5}dx, f(z)=\frac{1}{z^2+2z+5}, z^2+2z+5=0, z=-1\pm 2i, c_1=-1+2i\in D,$$

$$Rf(\mathrm{Re}^{i\theta})\to 0(R\to\infty), J=\oint_C f(z)dz=2\pi i\operatorname*{Res}_{z=c_1} f(z)dz=2\pi i\lim_{z\to c_1}(z-c_1)\times\frac{1}{z^2+2z+5}$$

$$=2\pi i\lim_{z\to c_1}\frac{1}{2z+2}=\frac{2\pi i}{2c_1+2}=\frac{\pi i}{c_1+1}=\frac{\pi}{2}$$

(2)

$$J=\int_{-\infty}^{\infty}\frac{2x^2}{x^4+1}dx, f(z)=\frac{2z^2}{z^4+1}, z^4+1=0, z^4=-1=e^{\pi i+2n\pi i}, z=e^{\frac{\pi i+2n\pi i}{4}},$$

$$z_0=e^{\frac{\pi i}{4}}=c_1\in D, z_1=e^{\frac{3\pi i}{4}}=c_2\in D, z_2=e^{\frac{5\pi i}{4}}, z_3=e^{\frac{7\pi i}{4}}, N=2$$

$$Rf(\mathrm{Re}^{i\theta})\to 0(R\to\infty), J=\oint_C f(z)dz=2\pi i\sum_{j=1}^{N}\operatorname*{Res}_{z=c_j} f(z)dz=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}(z-c_j)f(z)$$

$$=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}(z-c_j)f(z)=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}(z-c_j)\frac{2z^2}{z^4+1}=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\frac{2z^2+4z(z-c_j)}{4z^3}$$

$$=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\frac{1}{2z}=\pi i(\frac{1}{c_1}+\frac{1}{c_2})=\pi i(e^{-\frac{\pi i}{4}}+e^{-\frac{3\pi i}{4}})=\pi i(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i)=\sqrt{2}\pi$$

(3)

$$J=\int_{0}^{\infty}\frac{x^2+4}{x^4+6x^2+5}dx=\frac{1}{2}\int_{-\infty}^{\infty}\frac{x^2+4}{x^4+6x^2+5}dx, f(z)=\frac{z^2+4}{2(z^4+6z^2+5)}, z^4+6z^2+5=(z^2+1)(z^2+5)=0$$

$$z=\pm i,\pm\sqrt{5}i,\quad c_1=i\in D, c_2=\sqrt{5}i\in D, Rf(\mathrm{Re}^{i\theta})\to 0(R\to\infty), N=2,$$

$$J=\oint_C f(z)dz=2\pi i\sum_{j=1}^{N}\operatorname*{Res}_{z=c_j} f(z)dz=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}(z-c_j)f(z)=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\frac{(z-c_j)(z^2+4)}{2(z^4+6z^2+5)}$$

$$=\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\frac{(z^2+4)+2z(z-c_j)}{4z^3+12z}=\pi i(\frac{c_1^2+4}{4c_1^3+12c_1}+\frac{c_2^2+4}{4c_2^3+12c_2})=\pi i\{\frac{i^2+4}{4i^3+12i}+\frac{(\sqrt{5})^2i^2+4}{4(\sqrt{5})^3i^3+12\sqrt{5}i}\}$$

$$=\pi(\frac{-1+4}{-4+12}+\frac{-5+4}{-20\sqrt{5}+12\sqrt{5}})=\pi(\frac{3}{8}+\frac{-1}{-8\sqrt{5}})=\frac{1}{8}(3+\frac{\sqrt{5}}{5})\pi$$

(4)

$$J=\int_{-\infty}^{\infty}\frac{2x^2+3x+1}{x^4+5x^2+4}dx, f(z)=\frac{2z^2+3z+1}{z^4+5z^2+4}, z^4+5z^2+4=(z^2+1)(z^2+4)=0$$

$$z=\pm i,\pm 2i,\quad c_1=i\in D, c_2=2i\in D, Rf(\mathrm{Re}^{i\theta})\to 0(R\to\infty), N=2,$$

$$J=\oint_C f(z)dz=2\pi i\sum_{j=1}^{N}\mathop{\mathrm{Res}}_{z=c_j} f(z)dz=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}(z-c_j)f(z)=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\frac{(z-c_j)(2z^2+3z+1)}{z^4+5z^2+4}$$

$$=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\frac{(2z^2+3z+1)+(z-c_j)(4z+3)}{4z^3+10z}=2\pi i(\frac{2c_1^2+3c_1+1}{4c_1^3+10c_1}+\frac{2c_2^2+3c_2+1}{4c_2^3+10c_2})$$

$$=2\pi i(\frac{2i^2+3i+1}{4i^3+10i}+\frac{8i^2+6i+1}{32i^3+20i})=2\pi(\frac{-2+1+3i}{-4+10}+\frac{-8+1+6i}{-32+20})$$

$$=2\pi(\frac{-1+3i}{6}+\frac{-7+6i}{-12})=2\pi(\frac{7-2}{12}+\frac{i}{2}-\frac{i}{2})\pi=\frac{5}{6}\pi$$

(5)

$$J=\int_{-\infty}^{\infty}\frac{1}{x^8+1}dx, f(z)=\frac{1}{z^8+1}, z^8+1=0, z^8=-1=e^{\pi i+2n\pi i}, z=e^{\frac{\pi i+2n\pi i}{8}},$$

$$c_1=e^{\frac{\pi i}{8}}\in D, c_2=e^{\frac{3\pi i}{8}}\in D, c_3=e^{\frac{5\pi i}{8}}\in D, c_4=e^{\frac{7\pi i}{8}}\in D, Rf(\mathrm{Re}^{i\theta})\to 0(R\to\infty), N=4,$$

$$J=\oint_C f(z)dz=2\pi i\sum_{j=1}^{N}\mathop{\mathrm{Res}}_{z=c_j} f(z)dz=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}(z-c_j)f(z)=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\frac{(z-c_j)}{z^8+1}$$

$$=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\frac{1}{8z^7}=2\pi i\sum_{j=1}^{N}\frac{1}{8c_j^7}=2\pi i\sum_{j=1}^{N}\frac{c_j}{8c_j^8}=-\frac{\pi i}{4}\sum_{j=1}^{N}c_j=-\frac{\pi i}{4}(c_1+c_2+c_3+c_4)$$

$$=-\frac{\pi i}{4}(e^{\frac{\pi i}{8}}+e^{\frac{3\pi i}{8}}+e^{\frac{5\pi i}{8}}+e^{\frac{7\pi i}{8}})=-\frac{\pi i}{4}e^{\frac{\pi i}{8}}(1+e^{\frac{2\pi i}{8}}+e^{\frac{4\pi i}{8}}+e^{\frac{6\pi i}{8}})=-\frac{\pi i}{4}e^{\frac{\pi i}{8}}(1+e^{\frac{\pi i}{4}}+e^{\frac{\pi i}{2}}+e^{\frac{3\pi i}{4}})$$

$$=-\frac{\pi i}{4}(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8})(1+\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i+i-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i)=-\frac{\pi i}{4}(\sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}+i\sqrt{\frac{1-\cos\frac{\pi}{4}}{2}})(1+\sqrt{2}i+i)$$

$$=-\frac{\pi i}{4}(\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}+i\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}})(1+\sqrt{2}i+i)=-\frac{\pi i}{8}(\sqrt{2+\sqrt{2}}+i\sqrt{2-\sqrt{2}})(1+\sqrt{2}i+i)$$

$$=-\frac{\pi i}{8}\sqrt{2+\sqrt{2}}(1+i\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}})(1+\sqrt{2}i+i)=-\frac{\pi i}{8}\sqrt{2+\sqrt{2}}(1+i\sqrt{\frac{(2-\sqrt{2})^2}{2}})(1+\sqrt{2}i+i)$$

$$= -\frac{\pi i}{8}\sqrt{2+\sqrt{2}}\,(1+i\frac{2-\sqrt{2}}{\sqrt{2}})(1+\sqrt{2}i+i) = -\frac{\pi i}{8}\sqrt{2+\sqrt{2}}\,\{(1+i\sqrt{2})-i\}\,\{(1+i\sqrt{2})+i\}$$

$$= -\frac{\pi i}{8}\sqrt{2+\sqrt{2}}\,\{(1+i\sqrt{2})^2 - i^2\} = -\frac{\pi i}{8}\sqrt{2+\sqrt{2}}\,(1+2\sqrt{2}i-2+1) = \frac{\pi}{4}\sqrt{2+\sqrt{2}}\sqrt{2} = \frac{\sqrt{4+2\sqrt{2}}}{4}\pi$$

$$J = \frac{\sqrt{4+2\sqrt{2}}}{4}\pi = \frac{\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}}{4}\pi = \frac{\pi}{2\sqrt{2-\sqrt{2}}}$$

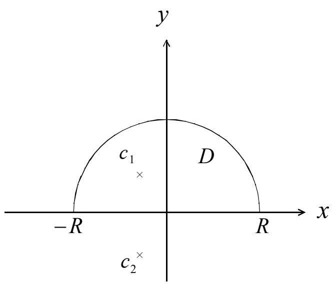

Fig.1 (1)

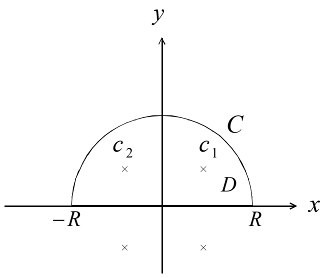

Fig.2 (2)

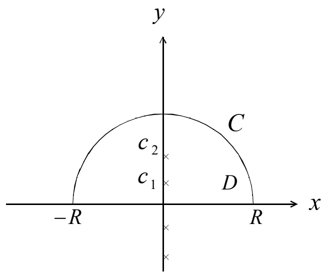

Fig.3 (3)

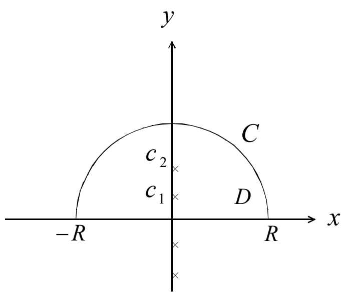

Fig.4 (4)

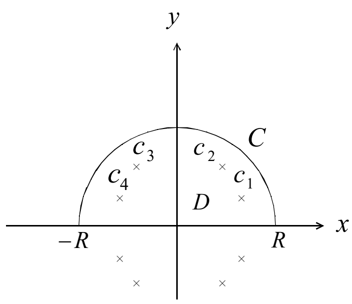

Fig.5 (5)