数学Ⅲ　レポート　No.7　　解答　　　　　　(2016)

1.　(20 点×5=100 点)

(1)

$$S=\int_{-\infty}^{\infty}\frac{\cos\pi x}{x^2+2x+2}dx,\ \ J=S+iT=\int_{-\infty}^{\infty}\frac{\cos\pi x}{x^2+2x+2}dx+i\int_{-\infty}^{\infty}\frac{\sin\pi x}{x^2+2x+2}dx=\int_{-\infty}^{\infty}\frac{e^{\pi xi}}{x^2+2x+2}dx,$$

$$f(z)=\frac{1}{z^2+2z+2},\ \ f(\mathrm{Re}^{i\theta})=\frac{1}{(\mathrm{Re}^{i\theta})^2+2\,\mathrm{Re}^{i\theta}+2}\to 0\quad(R\to\infty),$$

$$m=\pi>0, z^2+2z+2=0, z=-1\pm i, c_1=-1+i\in D,$$

$$J=\oint_C\frac{e^{\pi zi}}{z^2+2z+2}dz=2\pi i\,\mathop{\mathrm{Re}s}_{z=c_1}\left(\frac{e^{\pi zi}}{z^2+2z+2}\right)dz=2\pi i\lim_{z\to c_1}\frac{(z-c_1)e^{\pi zi}}{z^2+2z+2}=2\pi i\lim_{z\to c_1}\frac{e^{\pi zi}+(z-c_1)e^{\pi zi}\times\pi i}{2z+2}$$

$$=\frac{\pi i e^{\pi c_1 i}}{c_1+1}=\frac{\pi i e^{\pi(-1+i)i}}{-1+i+1}=\pi \mathrm{e}^{-\pi i-\pi}=-\pi e^{-\pi}, S=\mathrm{Re}(J)=-\pi e^{-\pi}$$

(2)

$$S=\int_{-\infty}^{\infty}\frac{x\cos\pi x}{x^2+4x+5}dx,\ \ J=S+iT=\int_{-\infty}^{\infty}\frac{x\cos\pi x}{x^2+4x+5}dx+i\int_{-\infty}^{\infty}\frac{\sin\pi x}{x^2+4x+5}dx=\int_{-\infty}^{\infty}\frac{xe^{\pi xi}}{x^2+4x+5}dx,$$

$$f(z)=\frac{z}{z^2+4z+5},\ \ f(\mathrm{Re}^{i\theta})\to 0\quad(R\to\infty), m=\pi>0, z^2+4z+5=0, z=-2\pm i, c_1=-2+i\in D,$$

$$J=\oint_C\frac{ze^{\pi zi}}{z^2+4z+5}dz=2\pi i\,\mathop{\mathrm{Re}s}_{z=c_1}\left(\frac{ze^{\pi zi}}{z^2+4z+5}\right)dz=2\pi i\lim_{z\to c_1}\frac{(z-c_1)ze^{\pi zi}}{z^2+4z+5}$$

$$=2\pi i\lim_{z\to c_1}\frac{(2z-c_1)e^{\pi zi}+(z-c_1)ze^{\pi zi}\times\pi i}{2z+4}=\frac{\pi i c_1 e^{\pi c_1 i}}{c_1+2}$$

$$=\frac{\pi i(-2+i)e^{\pi(-2+i)i}}{-2+i+2}=\pi(-2+i)e^{-2\pi i-\pi}=-2\pi e^{-\pi}+i\pi e^{-\pi}, S=\mathrm{Re}(J)=-2\pi e^{-\pi}$$

(3)

$$T=\int_{-\infty}^{\infty}\frac{x\sin\pi x}{x^2+4x+5}dx,\ \ J=S+iT=-2\pi e^{-\pi}+i\pi e^{-\pi}, T=\mathrm{Im}(J)=\pi e^{-\pi}$$

(4)

$$S=\int_{-\infty}^{\infty}\frac{\cos\pi x}{x^4+10x^2+9}dx,\ \ J=S+iT=\int_{-\infty}^{\infty}\frac{\cos\pi x}{x^4+10x^2+9}dx+i\int_{-\infty}^{\infty}\frac{\sin\pi x}{x^4+10x^2+9}dx=\int_{-\infty}^{\infty}\frac{e^{\pi xi}}{x^4+10x^2+9}dx,$$

$$f(z)=\frac{1}{z^4+10z^2+9},\ \ f(\mathrm{Re}^{i\theta})\to 0\quad(R\to\infty), m=\pi>0, z^4+10z^2+9=(z^2+9)(z^2+1), z=\pm 3i,\pm i,$$

$$\mathrm{Im}(z)>0, z=3i, i, c_1=i, c_2=3i, N=2$$

$$J=\oint_C \frac{e^{\pi z i}}{z^4+10z^2+9}dz=2\pi i\sum_{j=1}^{N}\operatorname*{Res}_{z=c_j}(\frac{e^{\pi z i}}{z^4+10z^2+9})dz=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\frac{(z-c_j)e^{\pi z i}}{z^4+10z^2+9}$$

$$=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\frac{e^{\pi z i}+(z-c_j)e^{\pi z i}\pi i}{4z^3+20z}=2\pi i(\frac{e^{\pi c_1 i}}{4c_1^3+20c_1}+\frac{e^{\pi c_2 i}}{4c_2^3+20c_2})$$

$$=2\pi i(\frac{e^{\pi i^2}}{4i^3+20i}+\frac{e^{\pi 3i^2}}{4(3i)^3+20(3i)})=2\pi(\frac{e^{-\pi}}{4i^2+20}+\frac{e^{-3\pi}}{27\times 4i^2+20\times 3})$$

$$=2\pi(\frac{e^{-\pi}}{16}+\frac{e^{-3\pi}}{60-108})=\pi(\frac{e^{-\pi}}{8}-\frac{e^{-3\pi}}{24})=\frac{\pi}{8}(e^{-\pi}-\frac{e^{-3\pi}}{3}),S=\operatorname{Re}(J)=\frac{\pi}{8}(e^{-\pi}-\frac{e^{-3\pi}}{3})$$

(5)

$$T=\int_{-\infty}^{\infty}\frac{x\sin\pi x}{x^4+5x^2+4}dx,\ \ J=S+iT=\int_{-\infty}^{\infty}\frac{x\cos\pi x}{x^4+5x^2+4}dx+i\int_{-\infty}^{\infty}\frac{x\sin\pi x}{x^4+5x^2+4}dx=\int_{-\infty}^{\infty}\frac{xe^{\pi x i}}{x^4+5x^2+4}dx,$$

$$f(z)=\frac{z}{z^4+5z^2+4},\ \ f(\mathrm{Re}^{i\theta})\to 0\ \ (R\to\infty),m=\pi>0,z^4+5z^2+4=(z^2+4)(z^2+1),z=\pm 2i,\pm i,$$

$$\operatorname{Im}(z)>0,z=2i,i,c_1=i,c_2=2i,N=2$$

$$J=\oint_C \frac{ze^{\pi z i}}{z^4+5z^2+4}dz=2\pi i\sum_{j=1}^{N}\operatorname*{Res}_{z=c_j}(\frac{ze^{\pi z i}}{z^4+5z^2+4})dz=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\frac{(z-c_j)ze^{\pi z i}}{z^4+5z^2+4}$$

$$=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\frac{(2z-c_j)e^{\pi z i}+(z-c_j)ze^{\pi z i}\pi i}{4z^3+10z}=2\pi i(\frac{c_1e^{\pi c_1 i}}{4c_1^3+10c_1}+\frac{c_2e^{\pi c_2 i}}{4c_2^3+10c_2})$$

$$=2\pi i(\frac{ie^{\pi i^2}}{4i^3+10i}+\frac{2ie^{\pi 2i^2}}{4(2i)^3+10(2i)})=2\pi i(\frac{e^{-\pi}}{4i^2+10}+\frac{2e^{-2\pi}}{32i^2+20})=2\pi i(\frac{e^{-\pi}}{6}-\frac{e^{-2\pi}}{6})$$

$$=\pi(\frac{e^{-\pi}}{3}-\frac{e^{-2\pi}}{3})i=\frac{\pi}{3}(e^{-\pi}-e^{-2\pi})i,T=\operatorname{Im}(J)=\frac{\pi}{3}(e^{-\pi}-e^{-2\pi})$$

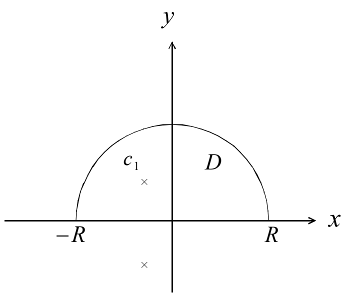

*Fig*1 (1)

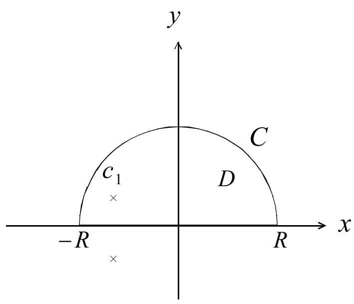

Fig.2 (2)

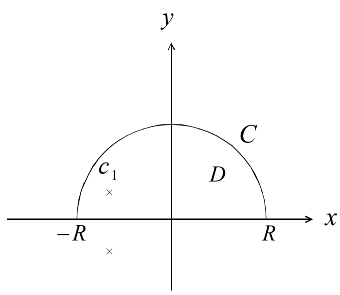

Fig .3 (3)

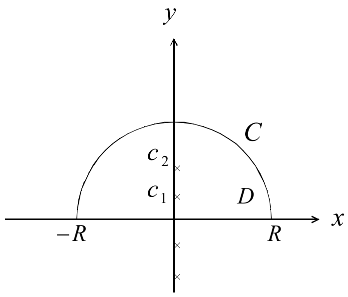

Fig.4 (4)

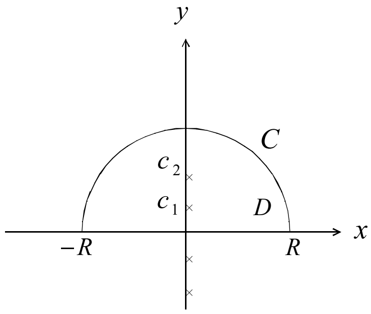

Fig.5 (5)