1.　(10 点×5=50 点)

(1)

$$J=\int_0^{2\pi}\frac{1}{6+5\cos\theta}d\theta,\ J=\int_0^{2\pi}\frac{1}{6+5\times\dfrac{e^{i\theta}+e^{-i\theta}}{2}}d\theta,\quad e^{i\theta}=z, e^{i\theta}id\theta=dz, d\theta=\frac{dz}{zi},$$

$$J=\int_{|z|=1}\frac{\dfrac{dz}{zi}}{6+\dfrac{5}{2}(z+z^{-1})}=\int_{|z|=1}\frac{2dz}{zi(12+5z+5z^{-1})}=\frac{2}{i}\int_{|z|=1}\frac{dz}{12z+5z^2+5}=\frac{2}{i}\int_{|z|=1}\frac{dz}{5z^2+12z+5},$$

$$5z^2+12z+5=0, z=\frac{-6\pm\sqrt{36-25}}{5}=\frac{-6\pm\sqrt{11}}{5}, c_1=\frac{-6+\sqrt{11}}{5}\in D,$$

$$J=2\pi i\operatorname*{Res}_{z=c_1}\left(\frac{2}{i}\times\frac{1}{5z^2+12z+5}\right)dz=2\pi i\lim_{z\to c_1}\frac{2(z-c_1)}{i(5z^2+12z+5)}=4\pi\lim_{z\to c_1}\frac{1}{10z+12}=\frac{4\pi}{10c_1+12}$$

$$=\frac{4\pi}{10\times\dfrac{\sqrt{11}-6}{5}+12}=\frac{4\pi}{2\sqrt{11}}=\frac{2\sqrt{11}}{11}\pi$$

(2)

$$J=\int_0^{2\pi}\frac{\sin\theta}{6+5\cos\theta}d\theta,\ J=\int_{|z|=1}\frac{\dfrac{z-z^{-1}}{2i}}{6+\dfrac{5}{2}(z+z^{-1})}\times\frac{dz}{zi}=\frac{1}{2i^2}\int_{|z|=1}\frac{(z-z^{-1})dz}{6z+\dfrac{5}{2}(z^2+1)}=-\int_{|z|=1}\frac{(z^2-1)dz}{z(12z+5z^2+5)}$$

$$=-\int_{|z|=1}\frac{(z^2-1)dz}{z(5z^2+12z+5)}, 5z^2+12z+5=0, z=\frac{-6\pm\sqrt{36-25}}{5}=\frac{-6\pm\sqrt{11}}{5},$$

$$c_1=\frac{-6+\sqrt{11}}{5}\in D, c_2=0\in D, N=2, J=2\pi i\sum_{j=1}^{N}\operatorname*{Res}_{z=c_j}\left\{-\frac{(z^2-1)}{z(5z^2+12z+5)}\right\}dz$$

$$=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\left\{-\frac{(z-c_j)(z^2-1)}{5z^3+12z^2+5z}\right\}=2\pi i\sum_{j=1}^{N}\lim_{z\to c_j}\left\{-\frac{(z^2-1)+(z-c_j)\times 2z}{15z^2+24z+5}\right\}$$

$$=2\pi i\sum_{j=1}^{N}-\frac{c_j^2-1}{15c_j^2+24c_j+5}=2\pi i\left(-\frac{c_1^2-1}{15c_1^2+24c_1+5}-\frac{-1}{5}\right), 15z^2+24z+5=3(5z^2+12z+5)-12z-10$$

$$=2\pi i\left(\frac{1}{5}-\frac{c_1^2-1}{-12c_1-10}\right)=2\pi i\left(\frac{1}{5}+\frac{c_1^2-1}{12c_1+10}\right)=2\pi i\left(\frac{1}{5}+\frac{-\dfrac{12}{5}c_1-1-1}{12c_1+10}\right)=2\pi i\left\{\frac{1}{5}-\frac{12c_1+10}{5(12c_1+10)}\right\}=0$$

(3)

$$J=\int_0^{2\pi}\frac{1}{8+7\cos\theta}d\theta,\ J=\int_{|z|=1}\frac{1}{8+\frac{7}{2}(z+z^{-1})}\times\frac{dz}{zi}=\frac{2}{i}\int_{|z|=1}\frac{dz}{16z+7(z^2+1)}=\frac{2}{i}\int_{|z|=1}\frac{dz}{7z^2+16z+7},$$

$$7z^2+16z+7=0, z=\frac{-8\pm\sqrt{64-49}}{7}=\frac{-8\pm\sqrt{15}}{7}, c_1=\frac{-8+\sqrt{15}}{7}\in D, c_2=\frac{-8-\sqrt{15}}{7}\notin D, N=1,$$

$$J=2\pi i\times\frac{2}{i}\times\lim_{z\to c_1}\frac{z-c_1}{7z^2+16z+7}=4\pi\times\lim_{z\to c_1}\frac{1}{14z+16}=\frac{4\pi}{14c_1+16}=\frac{2\pi}{7c_1+8}=\frac{2\pi}{7\times\frac{-8+\sqrt{15}}{7}+8}$$

$$=\frac{2\pi}{\sqrt{15}}=\frac{2\sqrt{15}}{15}\pi$$

(4)

$$J=\int_0^{2\pi}\frac{2+\sin\theta}{4+3\cos\theta}d\theta,\ J=\int_{|z|=1}\frac{2+\frac{z-z^{-1}}{2i}}{4+\frac{3}{2}(z+z^{-1})}\times\frac{dz}{zi}=-\int_{|z|=1}\frac{4i+z-z^{-1}}{8z+3(z^2+1)}dz=-\int_{|z|=1}\frac{z^2+4iz-1}{(3z^2+8z+3)z}dz,$$

$$3z^2+8z+3=0, z=\frac{-4\pm\sqrt{16-9}}{3}=\frac{-4\pm\sqrt{7}}{3}, c_1=\frac{-4+\sqrt{7}}{3}\in D, \frac{-4-\sqrt{7}}{3}\notin D, c_2=0, N=2,$$

$$J=2\pi i\times(-1)\sum_{j=1}^{N}\operatorname{Res}_{z=c_j}\{\frac{z^2+4iz-1}{(3z^2+8z+3)z}\}dz=2\pi i\times(-1)\sum_{j=1}^{N}\lim_{z\to c_j}\{\frac{(z-c_j)(z^2+4iz-1)}{3z^3+8z^2+3z}\}$$

$$=2\pi i\times(-1)\sum_{j=1}^{N}\lim_{z\to c_j}\{\frac{(z^2+4iz-1)+(z-c_j)(2z+4i)}{9z^2+16z+3}\}=2\pi i\times(-1)\sum_{j=1}^{N}\frac{c_j^2+4ic_j-1}{9c_j^2+16c_j+3}$$

$$=-2\pi i(\frac{c_1^2+4ic_1-1}{9c_1^2+16c_1+3}-\frac{1}{3})=-\frac{2\pi i}{3}\frac{-6c_1^2-16c_1-6+12ic_1}{9c_1^2+16c_1+3}=\frac{8\pi c_1}{9c_1^2+16c_1+3},$$

$3c_1^2+8c_1+3=0$ より $9c_1^2+16c_1+3=-3(8c_1+3)+16c_1+3=-8c_1-6$

$$J=-\frac{8\pi c_1}{8c_1+6}=-\frac{4\pi c_1}{4c_1+3}=-\frac{4\pi\times\frac{-4+\sqrt{7}}{3}}{4\times\frac{-4+\sqrt{7}}{3}+3}=-\frac{4\pi(-4+\sqrt{7})}{-16+4\sqrt{7}+9}=\frac{4\pi(4-\sqrt{7})}{4\sqrt{7}-7}=\frac{4\pi(4-\sqrt{7})}{\sqrt{7}(4-\sqrt{7})}$$

$$=\frac{4\pi}{\sqrt{7}}=\frac{4\sqrt{7}}{7}\pi$$

(5)

$$J=\int_0^{2\pi}\frac{3+\cos\theta}{6+3\sin\theta}d\theta,\ \ J=\int_{|z|=1}\frac{3+\dfrac{z+z^{-1}}{2}}{6+\dfrac{3}{2i}(z-z^{-1})}\times\frac{dz}{zi}=\int_{|z|=1}\frac{6z+z^2+1}{\{12iz+3(z^2-1)\}z}dz=\int_{|z|=1}\frac{z^2+6z+1}{3(z^3+4iz^2-z)}dz,$$

$$(z^2+4iz-1)z=0, z=-2i\pm\sqrt{-4+1}=(-2\pm\sqrt{3})i, c_1=(-2+\sqrt{3})i\in D, (-2-\sqrt{3})i\notin D, c_2=0\in D, N=2,$$

$$J=\frac{2\pi i}{3}\sum_{j=1}^{N}\operatorname{Re}s_{z=c_j}\{\frac{z^2+6z+1}{(z^3+4iz^2-z)}\}dz=\frac{2\pi i}{3}\sum_{j=1}^{N}\lim_{z\to c_j}\{\frac{(z-c_j)(z^2+6z+1)}{(z^3+4iz^2-z)}\}$$

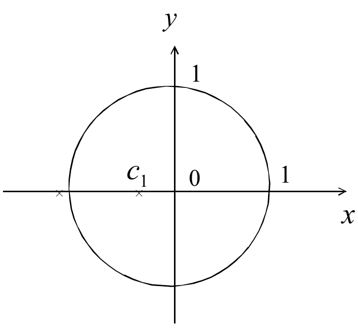
$$=\frac{2\pi i}{3}\sum_{j=1}^{N}\lim_{z\to c_j}\{\frac{(z^2+6z+1)+(z-c_j)(2z+6)}{3z^2+8iz-1}\}=\frac{2\pi i}{3}\sum_{j=1}^{N}\frac{c_j^2+6c_j+1}{3c_j^2+8ic_j-1}$$

$$=\frac{2\pi i}{3}(\frac{c_1^2+6c_1+1}{3c_1^2+8ic_1-1}-1)=\frac{2\pi i(-2c_1^2+6c_1-8ic_1+2)}{3(3c_1^2+8ic_1-1)}=\frac{-4\pi i(c_1^2+4ic_1-3c_1-1)}{3(3c_1^2+8ic_1-1)},$$

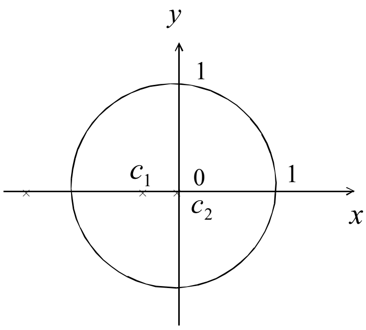
$c_1^2+4ic_1-1=0$ より $c_1^2=-4ic_1+1$

$$J=\frac{-4\pi i(-3c_1)}{3(-12ic_1+3+8ic_1-1)}=\frac{12\pi ic_1}{3(-4ic_1+2)}=\frac{2\pi ic_1}{-2ic_1+1}=\frac{2\pi i(-2+\sqrt{3})i}{1-2i(-2+\sqrt{3})i}=\frac{2\pi(2-\sqrt{3})}{1+2(-2+\sqrt{3})}=\frac{2\pi(2-\sqrt{3})}{2\sqrt{3}-3}$$

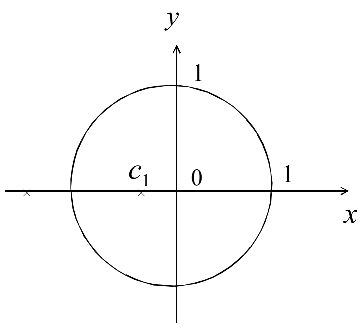
$$=\frac{2\pi}{\sqrt{3}}=\frac{2\sqrt{3}}{3}\pi$$



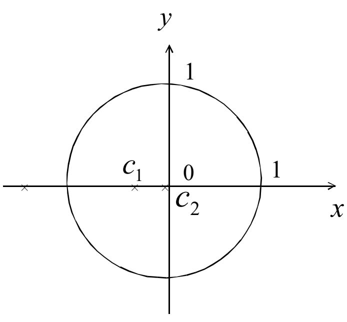
$Fig.1(1)$



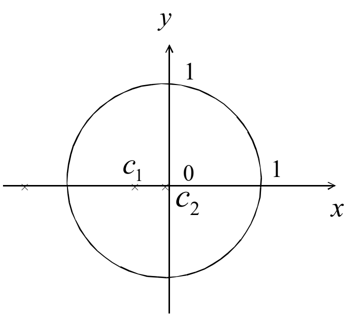
$Fig.2(2)$



$Fig.3(3)$



$Fig.4(4)$



$Fig.5(5)$

2. (10点×5＝50点)

(1)

$$f(z)=\frac{4z+3}{z^2+10z+21}=\frac{4z+3}{(z+3)(z+7)}=\frac{\alpha_1}{z+3}+\frac{\alpha_2}{z+7}$$

$$\alpha_1=\lim_{z\to(-3)}\frac{(z+3)(4z+3)}{(z+3)(z+7)}=\lim_{z\to(-3)}\frac{(4z+3)}{(z+7)}=-\frac{9}{4},\alpha_2=\lim_{z\to(-7)}\frac{(z+7)(4z+3)}{(z+3)(z+7)}=\lim_{z\to(-7)}\frac{(4z+3)}{(z+3)}=\frac{25}{4},$$

$$D=\{z\,|\,5<|z|<6\}$$

$$f(z)=-\frac{9}{4}\frac{1}{z+3}+\frac{25}{4}\frac{1}{z+7}=-\frac{9}{4}\frac{1}{z(1+\frac{3}{z})}+\frac{25}{4}\frac{1}{7(1+\frac{z}{7})}=-\frac{9}{4z}\sum_{n=0}^{\infty}(-\frac{3}{z})^n+\frac{25}{28}\sum_{n=0}^{\infty}(-\frac{z}{7})^n$$

$$=\frac{3}{4}\sum_{n=0}^{\infty}(-\frac{3}{z})^{n+1}+\frac{25}{28}\sum_{n=0}^{\infty}(-\frac{1}{7})^n z^n$$

$n+1=n', n'=n$と置き換える. $f(z)=\sum_{n=1}^{\infty}\frac{3}{4}(-3)^n z^{-n}+\sum_{n=0}^{\infty}\frac{25}{28}(-\frac{1}{7})^n z^n$

(2)

$$f(z)=\frac{2z+1}{z^2+7z+12}=\frac{2z+1}{(z+3)(z+4)}=\frac{\alpha_1}{z+3}+\frac{\alpha_2}{z+4}$$

$$\alpha_1=\lim_{z\to(-3)}\frac{(z+3)(2z+1)}{(z+3)(z+4)}=\lim_{z\to(-3)}\frac{(2z+1)}{(z+4)}=-5,\alpha_2=\lim_{z\to(-4)}\frac{(z+4)(2z+1)}{(z+3)(z+4)}=\lim_{z\to(-4)}\frac{(2z+1)}{(z+3)}=7,$$

$$D=\{z\,||z|<2\}$$

$$f(z)=\frac{-5}{z+3}+\frac{7}{z+4}=\frac{-5}{3(1+\frac{z}{3})}+\frac{7}{4(1+\frac{z}{4})}=-\frac{5}{3}\sum_{n=0}^{\infty}(-\frac{z}{3})^n+\frac{7}{4}\sum_{n=0}^{\infty}(-\frac{z}{4})^n=\sum_{n=0}^{\infty}\{\frac{7}{4}(-\frac{1}{4})^n-\frac{5}{3}(-\frac{1}{3})^n\}z^n$$

(3)

$$f(z)=\frac{3z^2+6z+2}{(z^2+2)(z+5)}=\frac{\alpha z+\beta}{z^2+2}+\frac{\gamma}{z+5}=\frac{(\alpha z+\beta)(z+5)+\gamma(z^2+2)}{(z^2+2)(z+5)}=\frac{(\alpha+\gamma)z^2+(5\alpha+\beta)z+(5\beta+2\gamma)}{(z^2+2)(z+5)}$$

$\alpha+\gamma=3, 5\alpha+\beta=6, 5\beta+2\gamma=2,$より$\gamma=3-\alpha, \beta=6-5\alpha, 5(6-5\alpha)+2(3-\alpha)=2, 36-27\alpha=2, \alpha=\frac{34}{27},$

$$\beta=6-\frac{34\times5}{27}=-\frac{8}{27},\gamma=3-\frac{34}{27}=\frac{47}{27}$$

$$D=\{z\,|\,2<|z|<4\}$$

$$f(z)=\frac{\frac{34}{27}z-\frac{8}{27}}{z^2+2}+\frac{\frac{47}{27}}{z+5}=\frac{\frac{34}{27}z-\frac{8}{27}}{z^2(1+\frac{2}{z^2})}+\frac{\frac{47}{27}}{5(1+\frac{z}{5})}=\frac{\frac{34}{27}z-\frac{8}{27}}{z^2}\sum_{n=0}^{\infty}(-\frac{2}{z^2})^n+\frac{47}{135}\sum_{n=0}^{\infty}(-\frac{z}{5})^n$$

$$=\sum_{n=0}^{\infty}\{\frac{34}{27}(-2)^n z^{-2n-1}-\frac{8}{27}(-2)^n z^{-2n-2}\}+\frac{47}{135}\sum_{n=0}^{\infty}(-\frac{z}{5})^n$$

$$f(z)=\sum_{n=0}^{\infty}\frac{34}{27}(-2)^n z^{-2n-1}-\sum_{n=0}^{\infty}\frac{8}{27}(-2)^n z^{-2n-2}+\sum_{n=0}^{\infty}\frac{47}{135}(-\frac{1}{5})^n z^n$$

(4)

$$f(z)=\frac{2z^2+5z+1}{(z^2+1)(z+3)}=\frac{\alpha z+\beta}{z^2+1}+\frac{\gamma}{z+3}=\frac{(\alpha z+\beta)(z+3)+\gamma(z^2+1)}{(z^2+1)(z+3)}=\frac{(\alpha+\gamma)z^2+(3\alpha+\beta)z+(3\beta+\gamma)}{(z^2+1)(z+3)}$$

$\alpha+\gamma=2, 3\alpha+\beta=5, 3\beta+\gamma=1,$ より $\gamma=2-\alpha, \beta=5-3\alpha, 3(5-3\alpha)+(2-\alpha)=1, 17-10\alpha=1, \alpha=\frac{8}{5},$

$\beta=5-\frac{24}{5}=\frac{1}{5}, \gamma=2-\frac{8}{5}=\frac{2}{5}$

$D=\{z\,\|z|>6\}$

$$f(z)=\frac{\frac{8}{5}z+\frac{1}{5}}{z^2+1}+\frac{\frac{2}{5}}{z+3}=\frac{\frac{8}{5}z+\frac{1}{5}}{z^2(1+\frac{1}{z^2})}+\frac{\frac{2}{5}}{z(1+\frac{3}{z})}=\frac{\frac{8}{5}z+\frac{1}{5}}{z^2}\sum_{n=0}^{\infty}(-\frac{1}{z^2})^n+\sum_{n=0}^{\infty}\frac{2}{5z}(-\frac{3}{z})^n$$

$$=\sum_{n=0}^{\infty}\{\frac{8}{5}(-1)^n z^{-2n-1}+\frac{1}{5}(-1)^n z^{-2n-2}\}+\sum_{n=0}^{\infty}\frac{2}{5}(-3)^n z^{-n-1}$$

$$=\sum_{n=0}^{\infty}\{\frac{8}{5}(-1)^n z^{-2n-1}+\frac{1}{5}(-1)^n z^{-2n-2}\}+\sum_{n=0}^{\infty}\{\frac{2}{5}(-3)^{2n} z^{-2n-1}+\frac{2}{5}(-3)^{2n+1} z^{-2n-2}\}$$

$$=\sum_{n=0}^{\infty}\{\frac{8}{5}(-1)^n+\frac{2}{5}(3)^{2n}\}z^{-2n-1}+\sum_{n=0}^{\infty}\{\frac{1}{5}(-1)^n-\frac{2}{5}(3)^{2n+1}\}z^{-2n-2}$$

(5)

$$f(z)=\frac{z^3+3z^2+4z+2}{z^4+11z^2+10}=\frac{z^3+3z^2+4z+2}{(z^2+10)(z^2+1)}=\frac{\alpha_1 z+\beta_1}{z^2+10}+\frac{\alpha_2 z+\beta_2}{z^2+1}$$

$$=\frac{(\alpha_1 z+\beta_1)(z^2+1)+(\alpha_2 z+\beta_2)(z^2+10)}{(z^2+10)(z^2+1)}$$

$$=\frac{(\alpha_1+\alpha_2)z^3+(\beta_1+\beta_2)z^2+(\alpha_1+10\alpha_2)z+(\beta_1+10\beta_2)}{(z^2+10)(z^2+1)},$$

$\alpha_1+\alpha_2=1, \beta_1+\beta_2=3, \alpha_1+10\alpha_2=4, \beta_1+10\beta_2=2,$ より

$\alpha_2=\frac{1}{3}, \alpha_1=\frac{2}{3}, \beta_2=-\frac{1}{9}, \beta_1=\frac{28}{9}$

$D=\{z\,|2<|z|<3\}$

$$f(z)=\frac{\frac{2}{3}z+\frac{28}{9}}{z^2+10}+\frac{\frac{1}{3}z-\frac{1}{9}}{z^2+1}=\frac{\frac{2}{3}z+\frac{28}{9}}{10(1+\frac{z^2}{10})}+\frac{\frac{1}{3}z-\frac{1}{9}}{z^2(1+\frac{1}{z^2})}=\frac{\frac{2}{3}z+\frac{28}{9}}{10}\sum_{n=0}^{\infty}(-\frac{z^2}{10})^n+\frac{\frac{1}{3}z-\frac{1}{9}}{z^2}\sum_{n=0}^{\infty}(-\frac{1}{z^2})^n$$

$$f(z)=\sum_{n=0}^{\infty}\{\frac{1}{15}(-\frac{1}{10})^n z^{2n+1}+\frac{14}{45}(-\frac{1}{10})^n z^{2n}\}+\sum_{n=0}^{\infty}\{\frac{1}{3}(-1)^n z^{-2n-1}-\frac{1}{9}(-1)^n z^{-2n-2}\}$$

2. (3),(4),(5) の別解

(3)

$$f(z)=\frac{3z^2+6z+2}{(z^2+2)(z+5)}=\frac{3z^2+6z+2}{(z+\sqrt{2}i)(z-\sqrt{2}i)(z+5)}=\frac{\alpha_1}{z+\sqrt{2}i}+\frac{\alpha_2}{z-\sqrt{2}i}+\frac{\alpha_3}{z+5}$$

$$\alpha_1=\lim_{z\to(-\sqrt{2}i)}\frac{(z+\sqrt{2}i)(3z^2+6z+2)}{(z^2+2)(z+5)}=\frac{17-2\sqrt{2}i}{27},\alpha_2=\lim_{z\to\sqrt{2}i}\frac{(z-\sqrt{2}i)(3z^2+6z+2)}{(z^2+2)(z+5)}=\frac{17+2\sqrt{2}i}{27},$$

$$\alpha_3=\lim_{z\to-5}\frac{(z+5)(3z^2+6z+2)}{(z^2+2)(z+5)}=\frac{47}{27},D=\{z\,|\,2<|z|<4\}$$

$$f(z)=\frac{\alpha_1}{z+\sqrt{2}i}+\frac{\alpha_2}{z-\sqrt{2}i}+\frac{\alpha_3}{z+5}=\frac{\alpha_1}{z(1+\frac{\sqrt{2}i}{z})}+\frac{\alpha_2}{z(1-\frac{\sqrt{2}i}{z})}+\frac{\alpha_3}{5(1+\frac{z}{5})}$$

$$=\frac{\alpha_1}{z}\sum_{n=0}^{\infty}(-\frac{\sqrt{2}i}{z})^n+\frac{\alpha_2}{z}\sum_{n=0}^{\infty}(\frac{\sqrt{2}i}{z})^n+\frac{\alpha_3}{5}\sum_{n=0}^{\infty}(-\frac{z}{5})^n$$

$$=\sum_{n=0}^{\infty}\{\alpha_1(-\sqrt{2}i)^n+\alpha_2(\sqrt{2}i)^n\}z^{-(n+1)}+\sum_{n=0}^{\infty}\frac{\alpha_3}{5}(-\frac{1}{5})^n z^n,n+1=n',$$

$$=\sum_{n'=1}^{\infty}\{\alpha_1(-\sqrt{2}i)^{n'-1}+\alpha_2(\sqrt{2}i)^{n'-1}\}z^{-n'}+\sum_{n=0}^{\infty}\frac{\alpha_3}{5}(-\frac{1}{5})^n z^n,n'\to n$$

$$=\sum_{n=1}^{\infty}\{\frac{17-2\sqrt{2}i}{27}(-\sqrt{2}i)^{n-1}+\frac{17+2\sqrt{2}i}{27}(\sqrt{2}i)^{n-1}\}z^{-n}+\sum_{n=0}^{\infty}\frac{47}{135}(-5)^{-n}z^n$$

(4)

$$f(z)=\frac{2z^2+5z+1}{(z^2+1)(z+3)}=\frac{2z^2+5z+1}{(z+i)(z-i)(z+3)}=\frac{\alpha_1}{z+i}+\frac{\alpha_2}{z-i}+\frac{\alpha_3}{z+3},$$

$$\alpha_1=\lim_{z\to-i}\frac{(z+i)(2z^2+5z+1)}{(z^2+1)(z+3)}=\frac{8+i}{10},\alpha_2=\lim_{z\to i}\frac{(z-i)(2z^2+5z+1)}{(z^2+1)(z+3)}=\frac{8-i}{10},$$

$$\alpha_3=\lim_{z\to-3}\frac{(z+3)(2z^2+5z+1)}{(z^2+1)(z+3)}=\frac{2}{5},D=\{z\,||z|>6\}$$

$$f(z)=\frac{\alpha_1}{z+i}+\frac{\alpha_2}{z-i}+\frac{\alpha_3}{z+3}=\frac{\alpha_1}{z(1+\frac{i}{z})}+\frac{\alpha_2}{z(1-\frac{i}{z})}+\frac{\alpha_3}{z(1+\frac{3}{z})}$$

$$=\sum_{n=0}^{\infty}\{\alpha_1(-i)^n+\alpha_2(i)^n+\alpha_3(-3)^n\}z^{-n-1},n+1=n'$$

$$f(z)=\sum_{n'=1}^{\infty}\{\alpha_1(-i)^{n'-1}+\alpha_2(i)^{n'-1}+\alpha_3(-3)^{n'-1}\}z^{-n'},n'\to n,$$

$$f(z)=\sum_{n=1}^{\infty}\{\frac{8+i}{10}(-i)^{n-1}+\frac{8-i}{10}(i)^{n-1}+\frac{2}{5}(-3)^{n-1}\}z^{-n}$$

(5)

$$f(z)=\frac{z^3+3z^2+4z+2}{z^4+11z^2+10}=\frac{z^3+3z^2+4z+2}{(z^2+10)(z^2+1)}=\frac{z^3+3z^2+4z+2}{(z+\sqrt{10}i)(z-\sqrt{10}i)(z+i)(z-i)},D=\{z|2<|z|<3\},$$

$$=\frac{\alpha_1}{z+\sqrt{10}i}+\frac{\alpha_2}{z-\sqrt{10}i}+\frac{\alpha_3}{z+i}+\frac{\alpha_4}{z-i},$$

$$\alpha_1=\lim_{z\to-\sqrt{10}i}\frac{(z+\sqrt{10}i)(z^3+3z^2+4z+2)}{(z^2+10)(z^2+1)}=\frac{30+14\sqrt{10}i}{90},$$

$$\alpha_2=\lim_{z\to\sqrt{10}i}\frac{(z-\sqrt{10}i)(z^3+3z^2+4z+2)}{(z^2+10)(z^2+1)}=\frac{30-14\sqrt{10}i}{90},$$

$$\alpha_3=\lim_{z\to-i}\frac{(z+i)(z^3+3z^2+4z+2)}{(z^2+10)(z^2+1)}=\frac{3-i}{18},\alpha_4=\lim_{z\to i}\frac{(z-i)(z^3+3z^2+4z+2)}{(z^2+10)(z^2+1)}=\frac{3+i}{18}$$

$$f(z)=\frac{\alpha_1}{\sqrt{10}i(1+\frac{z}{\sqrt{10}i})}-\frac{\alpha_2}{\sqrt{10}i(1-\frac{z}{\sqrt{10}i})}+\frac{\alpha_3}{z(1+\frac{i}{z})}+\frac{\alpha_4}{z(1-\frac{i}{z})}$$

$$f(z)=\sum_{n=0}^{\infty}\{\frac{\alpha_1}{\sqrt{10}i}(-\frac{1}{\sqrt{10}i})^n-\frac{\alpha_2}{\sqrt{10}i}(\frac{1}{\sqrt{10}i})^n\}z^n+\sum_{n=0}^{\infty}\{\alpha_3(-i)^n+\alpha_4 i^n\}z^{-n-1},n+1=n',$$

$$f(z)=\sum_{n=0}^{\infty}\{\frac{30+14\sqrt{10}i}{90\sqrt{10}i}(-\frac{1}{\sqrt{10}i})^n-\frac{30-14\sqrt{10}i}{90\sqrt{10}i}(\frac{1}{\sqrt{10}i})^n\}z^n+\sum_{n'=1}^{\infty}\{\frac{3-i}{18}(-i)^{n'-1}+\frac{3+i}{18}i^{n'-1}\}z^{-n'},n'\to n$$

$$f(z)=\sum_{n=0}^{\infty}\{\frac{14-3\sqrt{10}i}{90}(\frac{\sqrt{10}i}{10})^n+\frac{14+3\sqrt{10}i}{90}(-\frac{\sqrt{10}i}{10})^n\}z^n+\sum_{n=1}^{\infty}\{\frac{3-i}{18}(-i)^{n-1}+\frac{3+i}{18}i^{n-1}\}z^{-n}$$

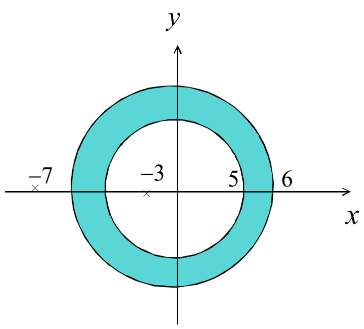


Fig.6(2−1)

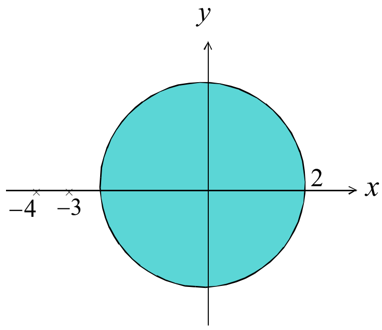


Fig.7(2−2)

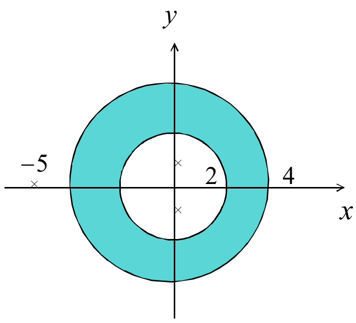


Fig.8(2−3)

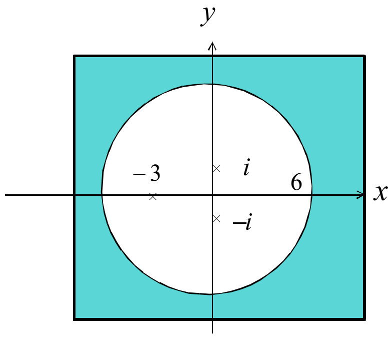


Fig .9(2 − 4)

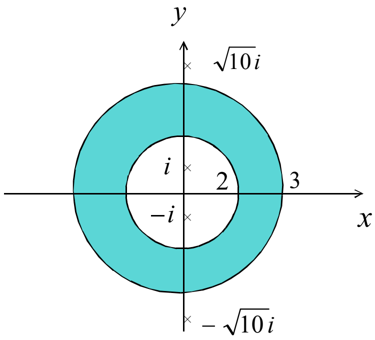


Fig .10 (2 − 5)